



*Meteorologisk
institutt
met.no*

On the effect of forward shear and reversed shear baroclinic flows for polar low developments

Thor Erik Nordeng
Norwegian Meteorological Institute

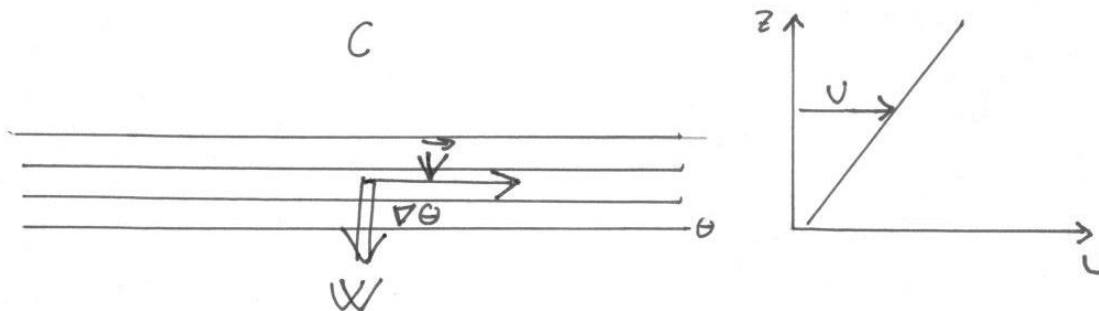


Outline

- Baroclinic growth
 - a) Normal mode solution
 - b) Initial value problem
 - c) The effect of friction and diabatic forcing
- Diabatic growth
 - a) CISK
 - b) WISHE
- Baroclinicity as a precursor for diabatic growth

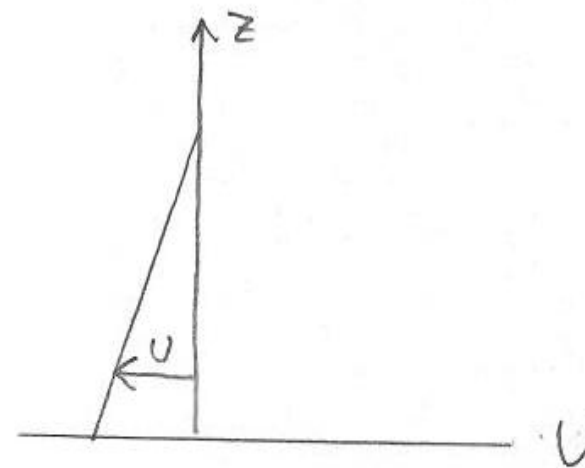
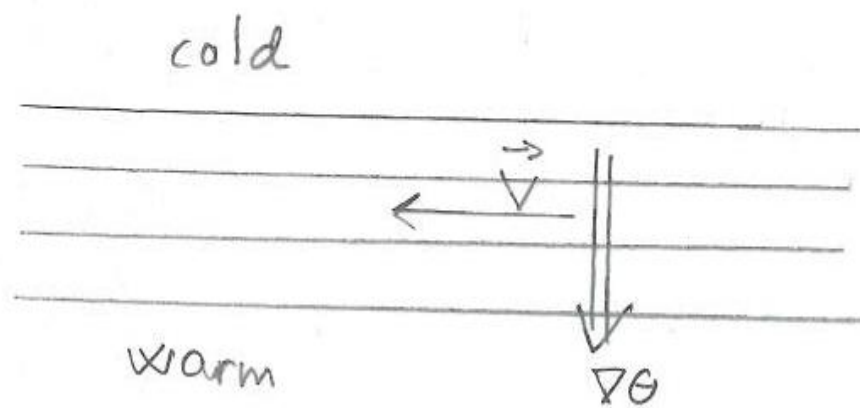


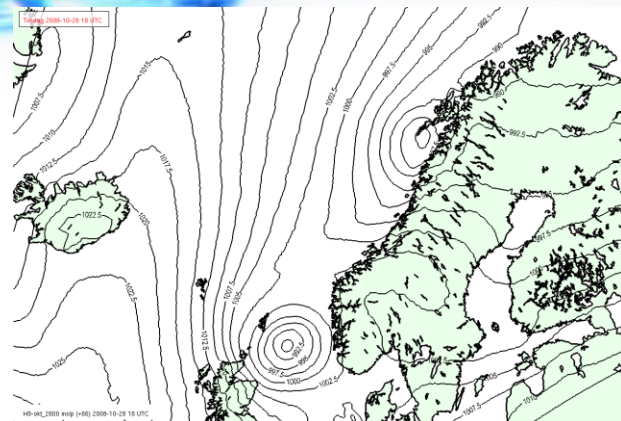
Forward shear



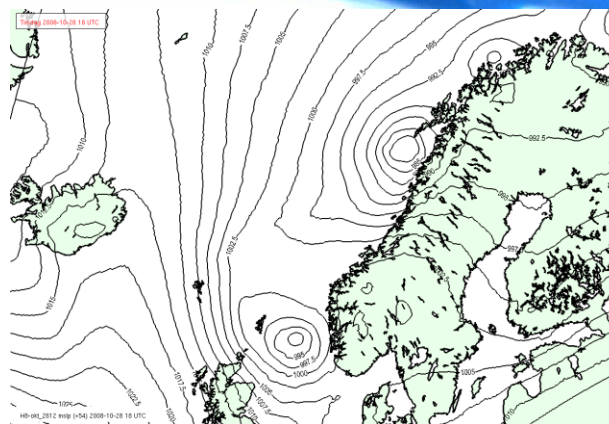


Reversed shear

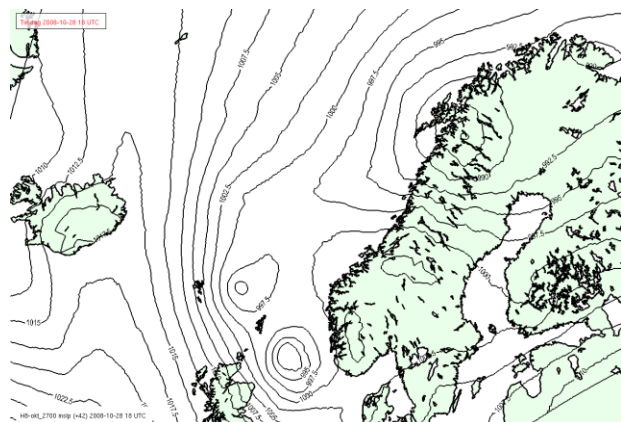




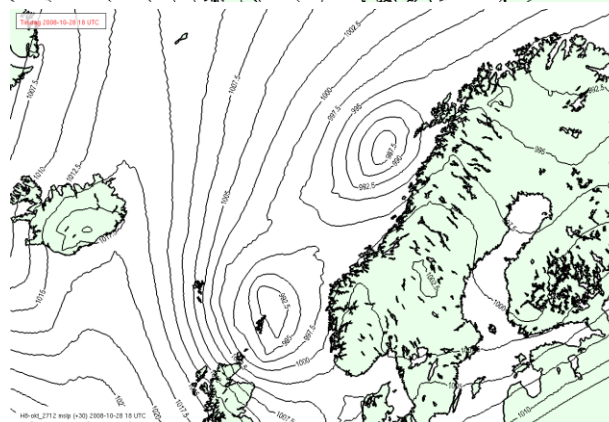
26 oct 2008
66hrs



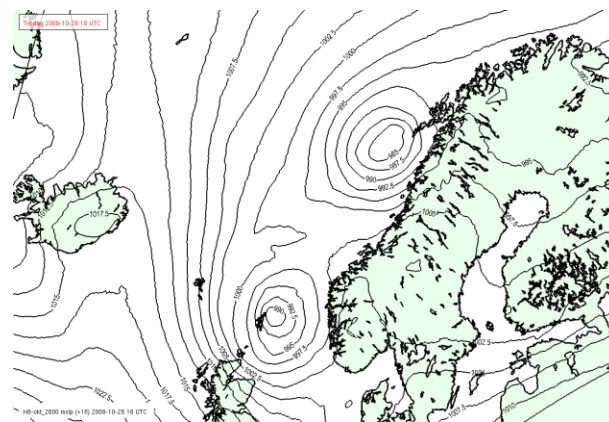
26 oct 2008
54 hrs



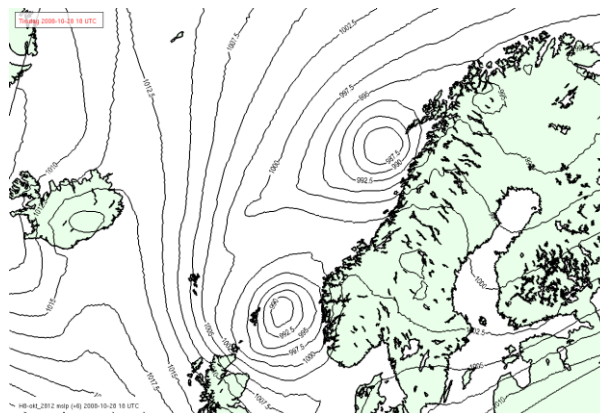
27 oct 2008
42 hrs



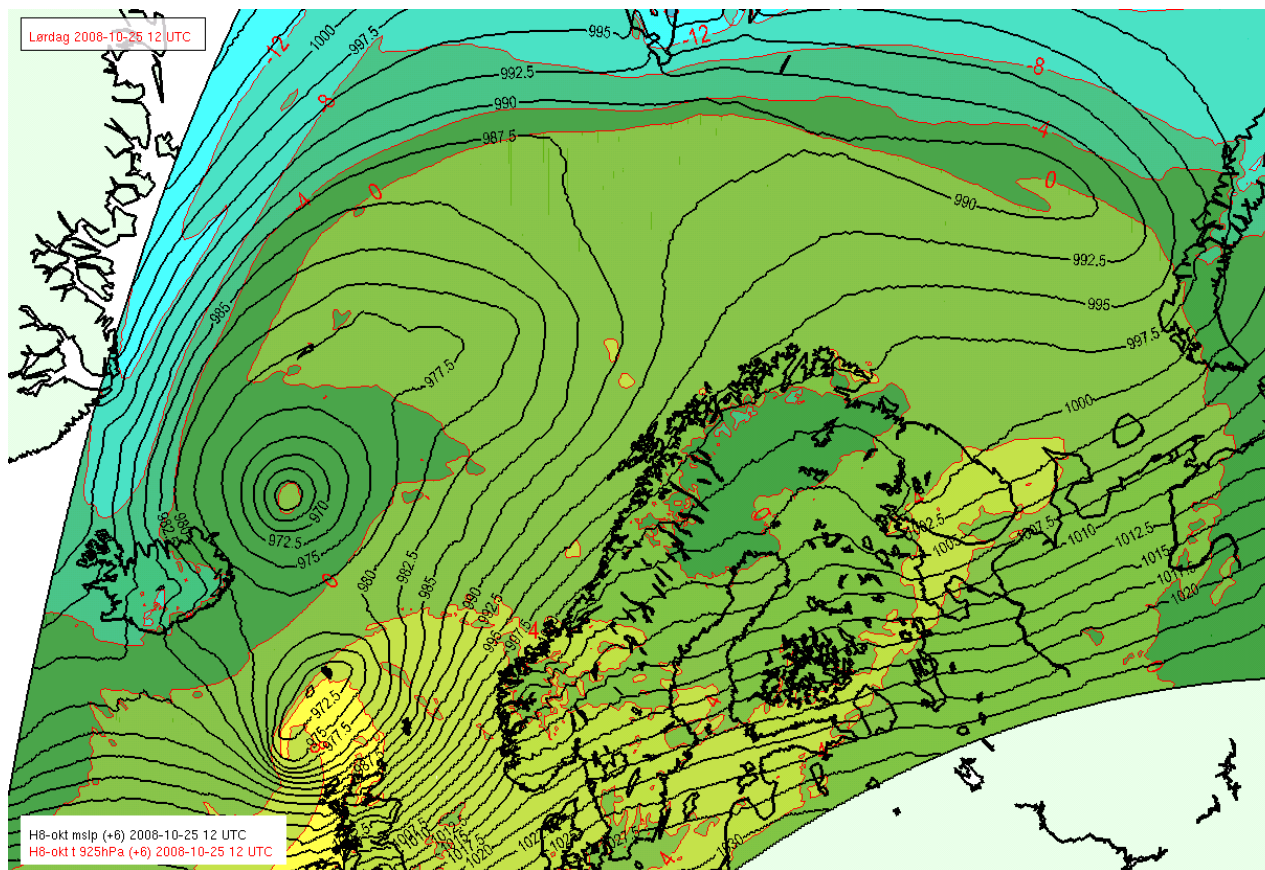
27 oct 2008
30 hrs

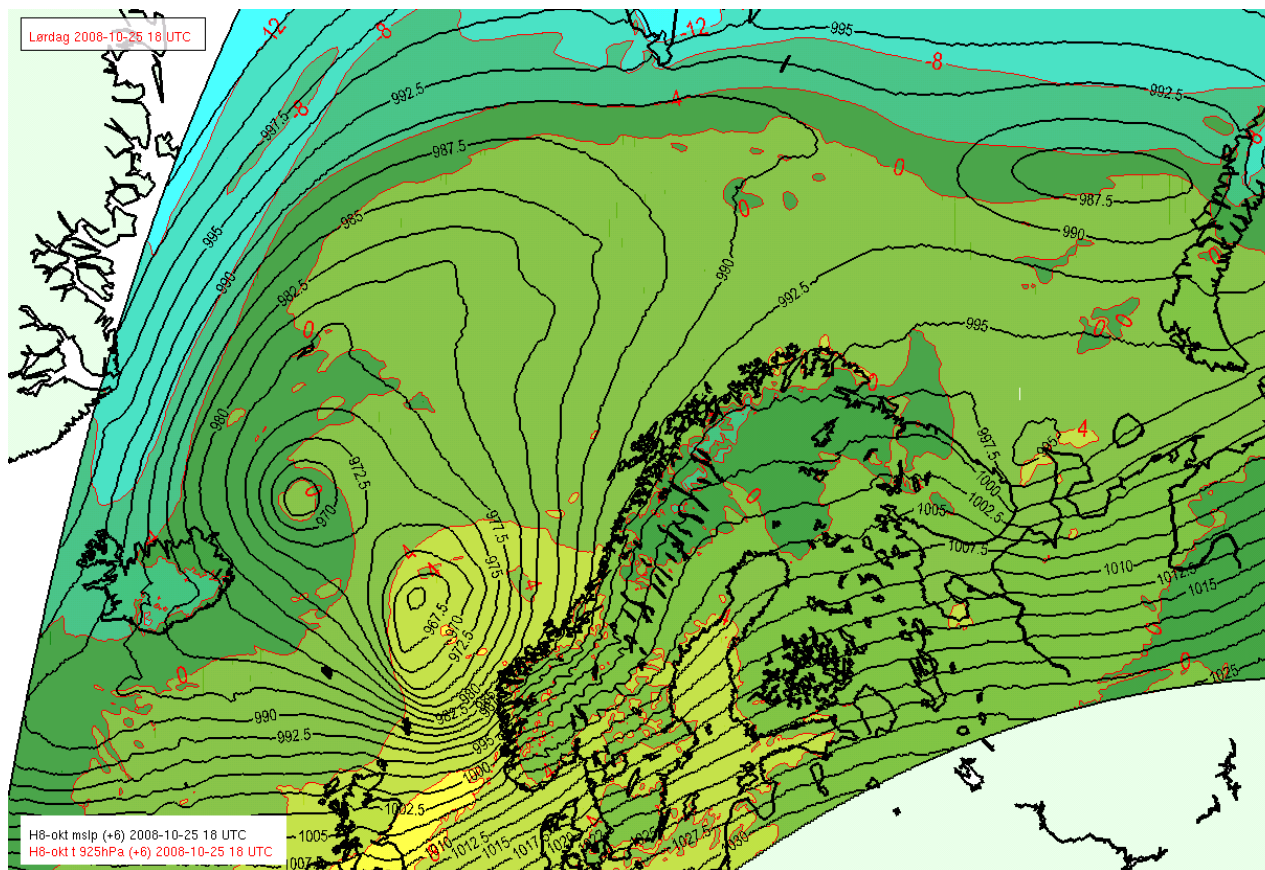


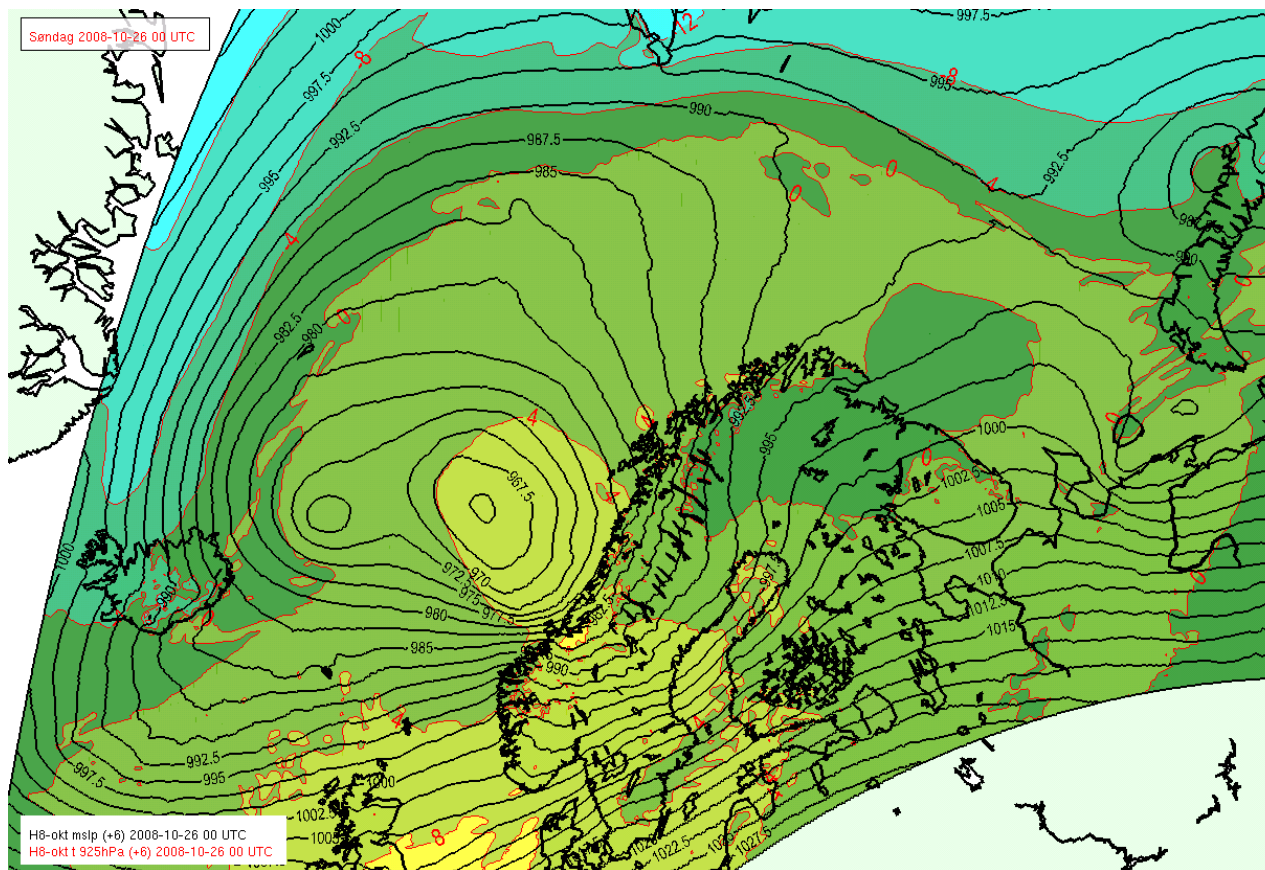
28 oct 2008
18 hrs

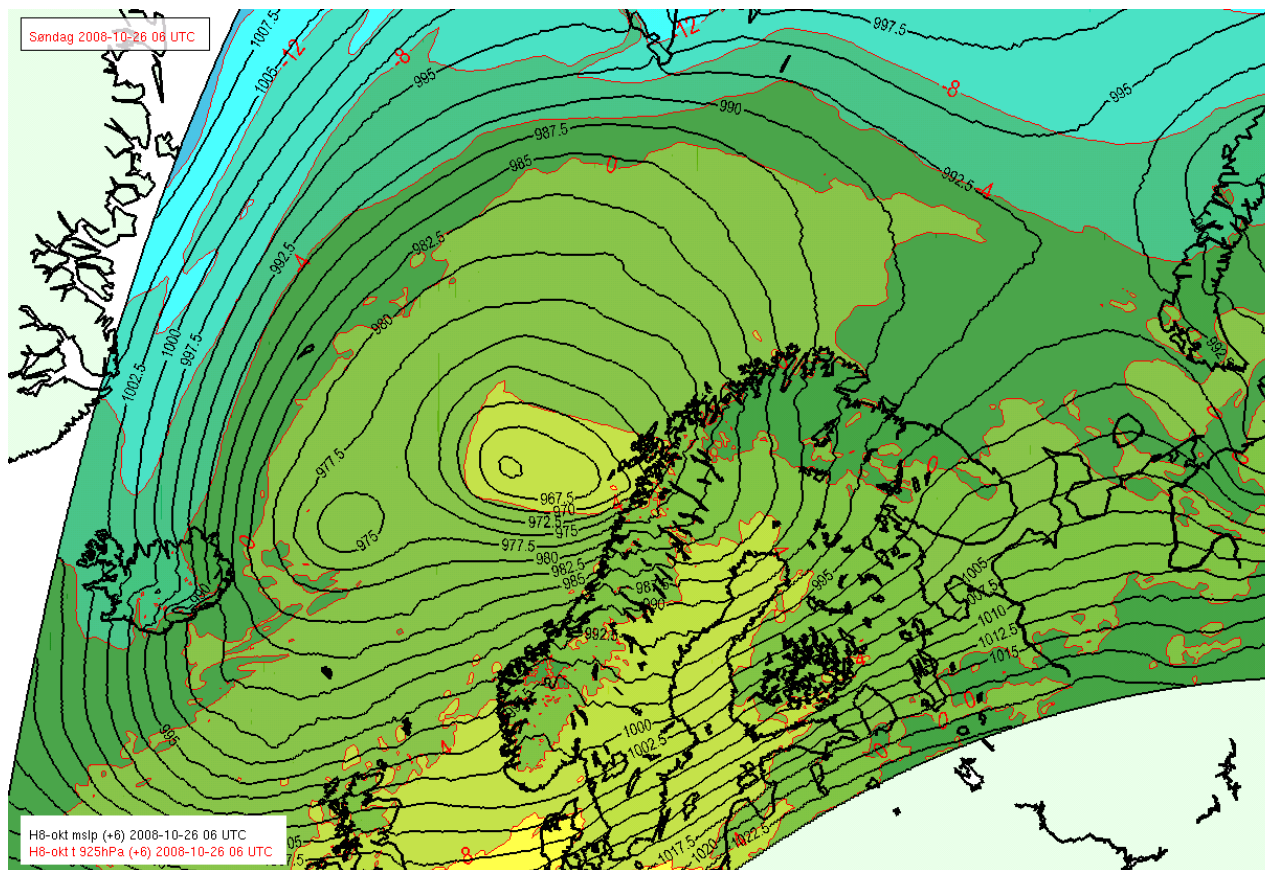


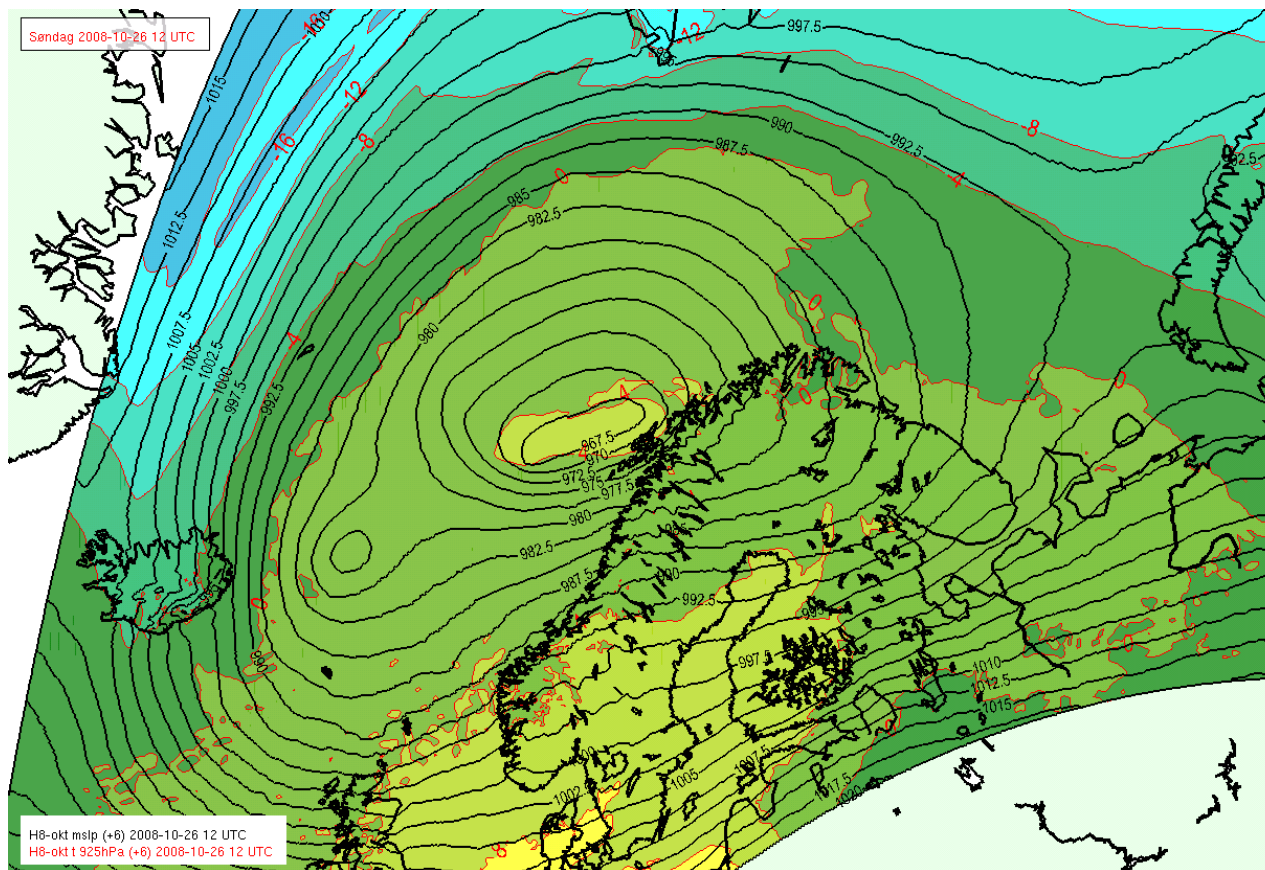
28 oct 2008
06 hrs

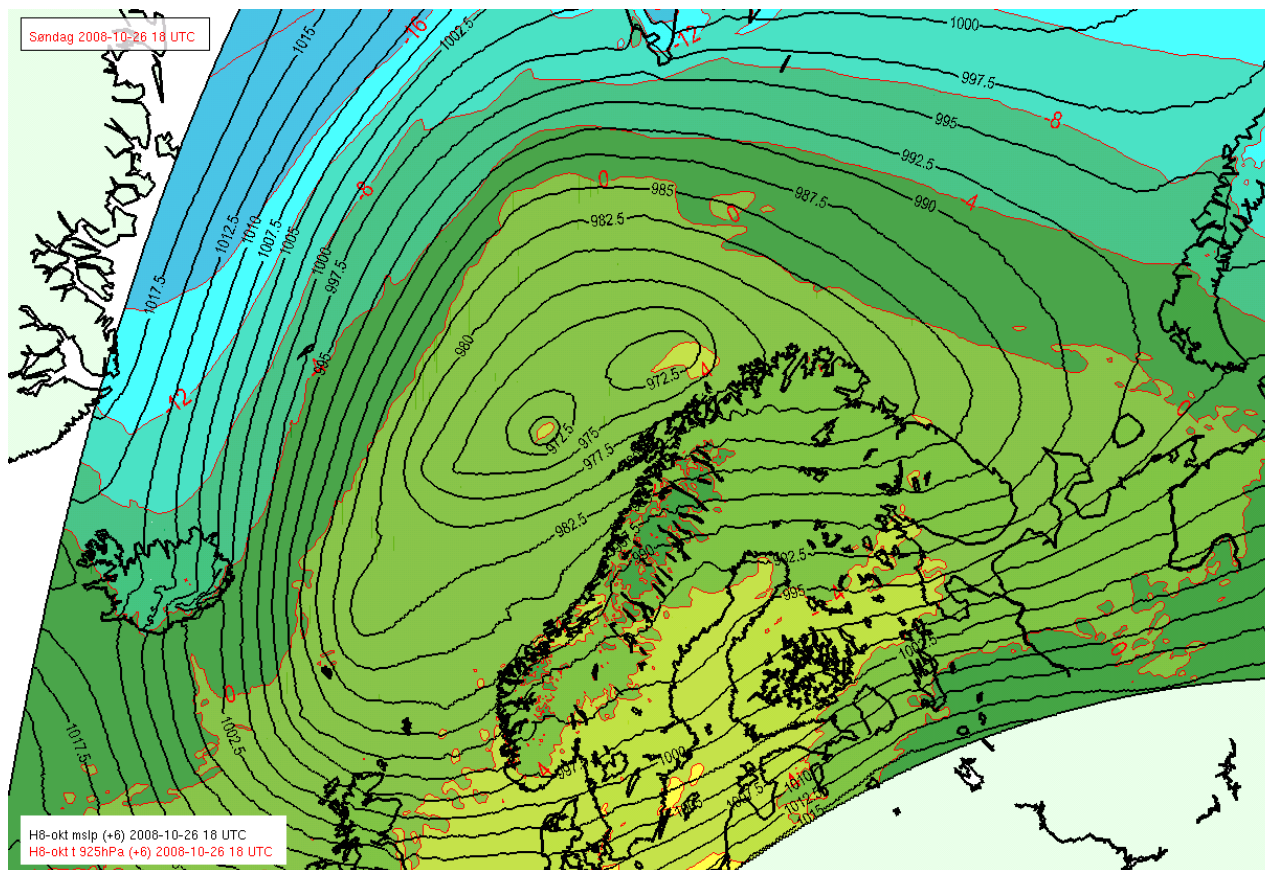


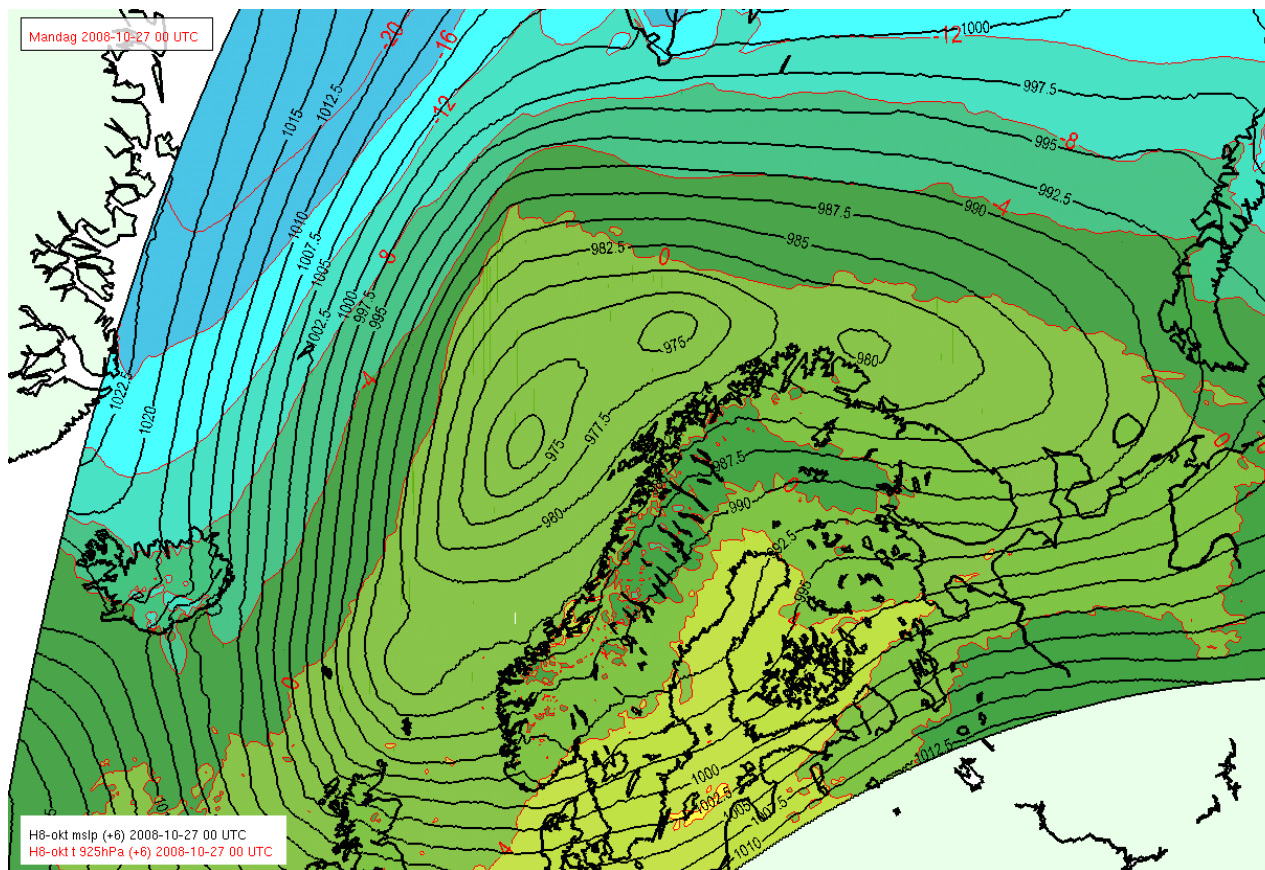


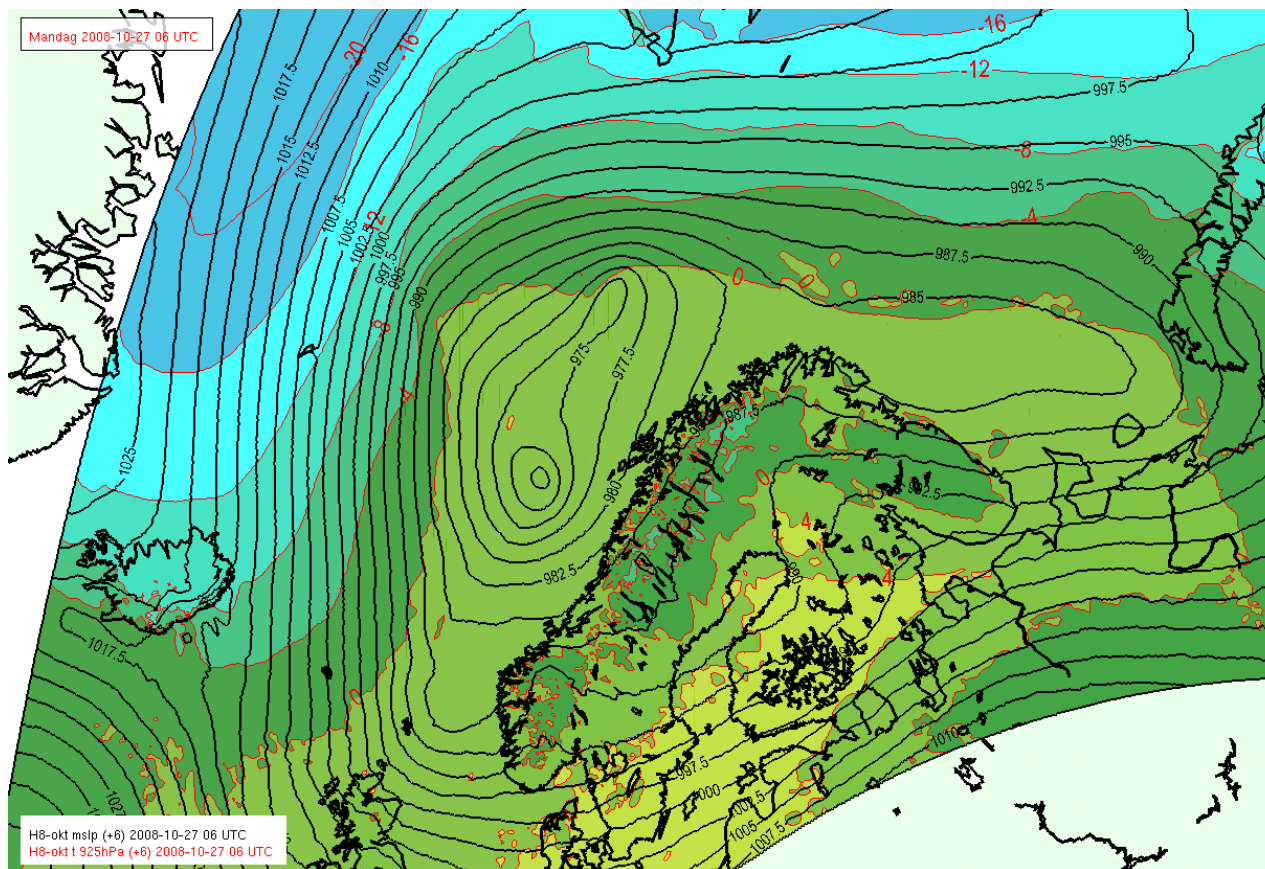


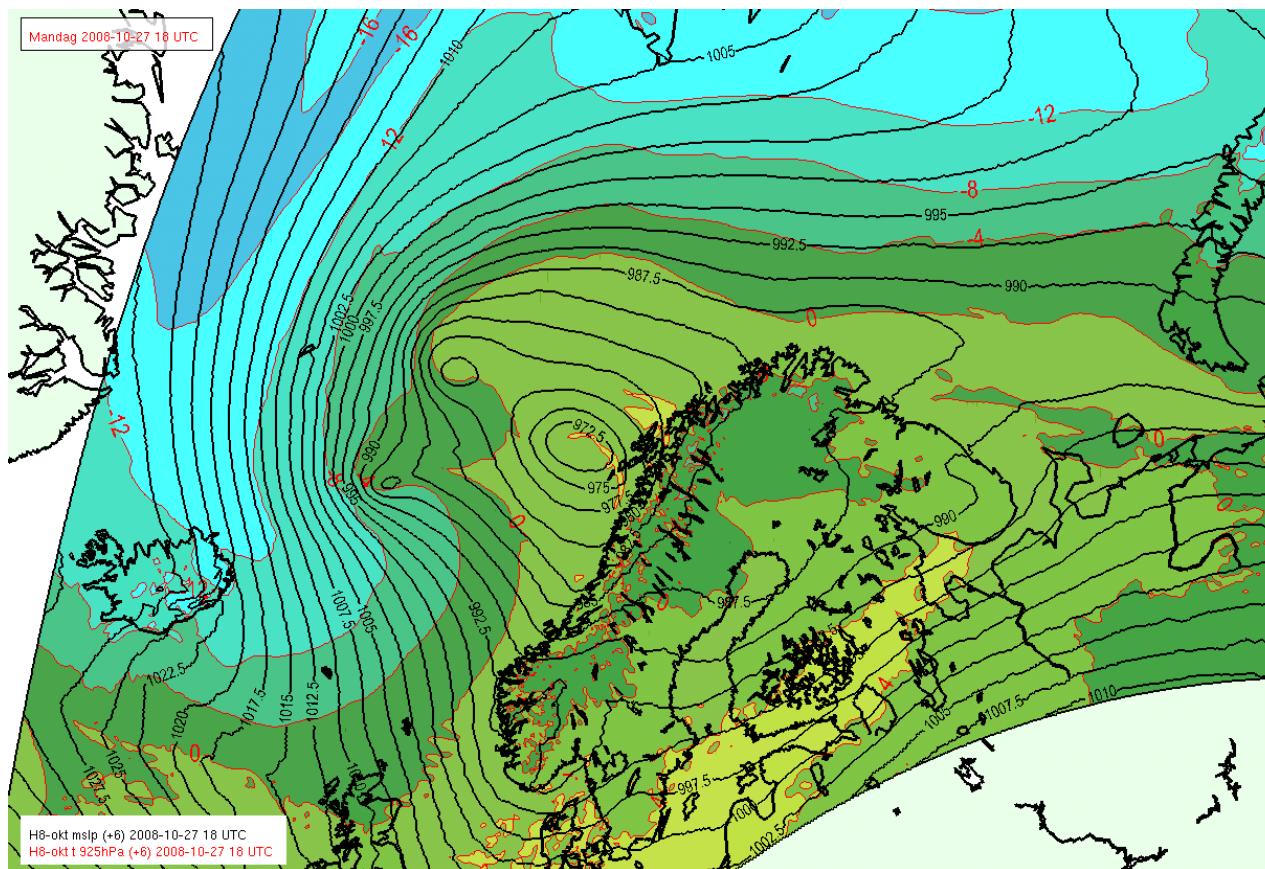


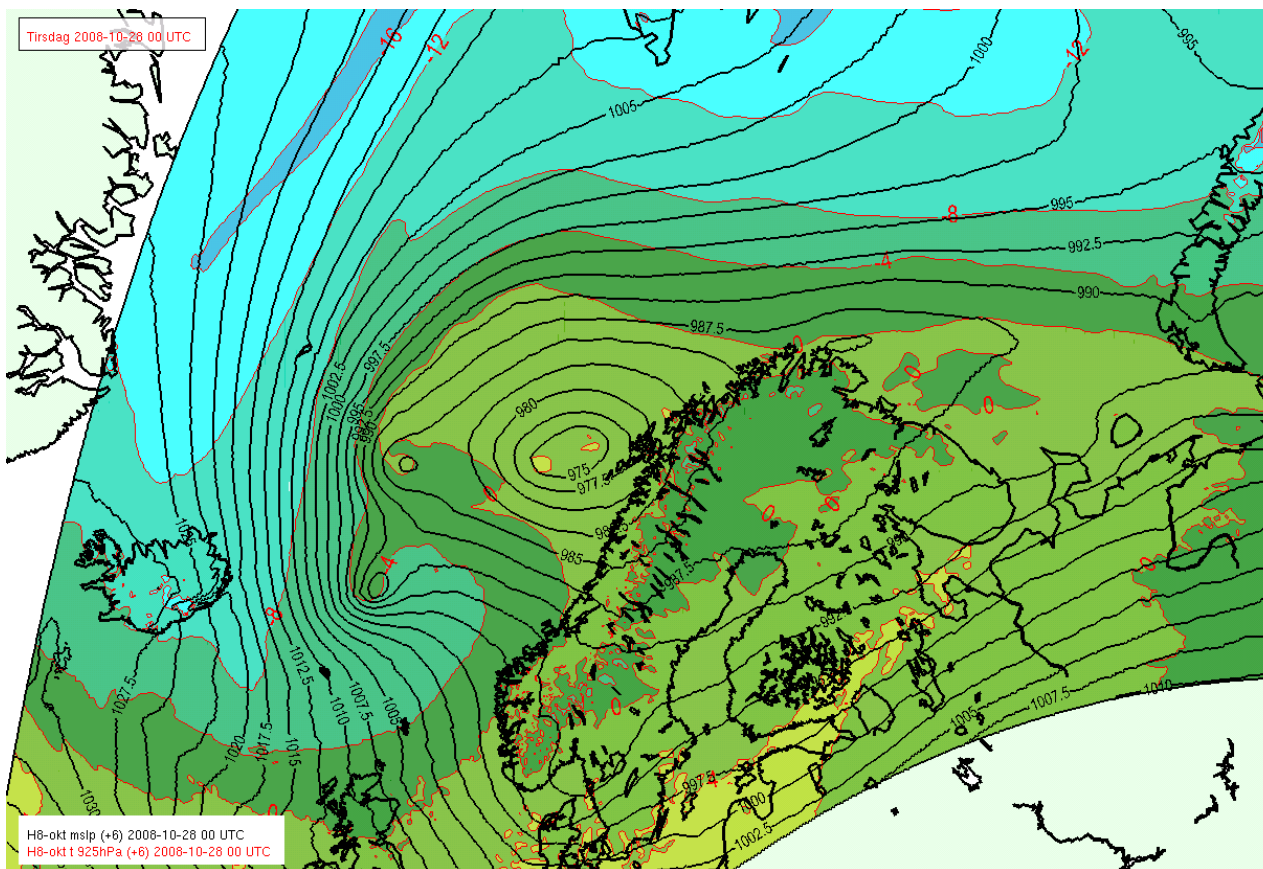


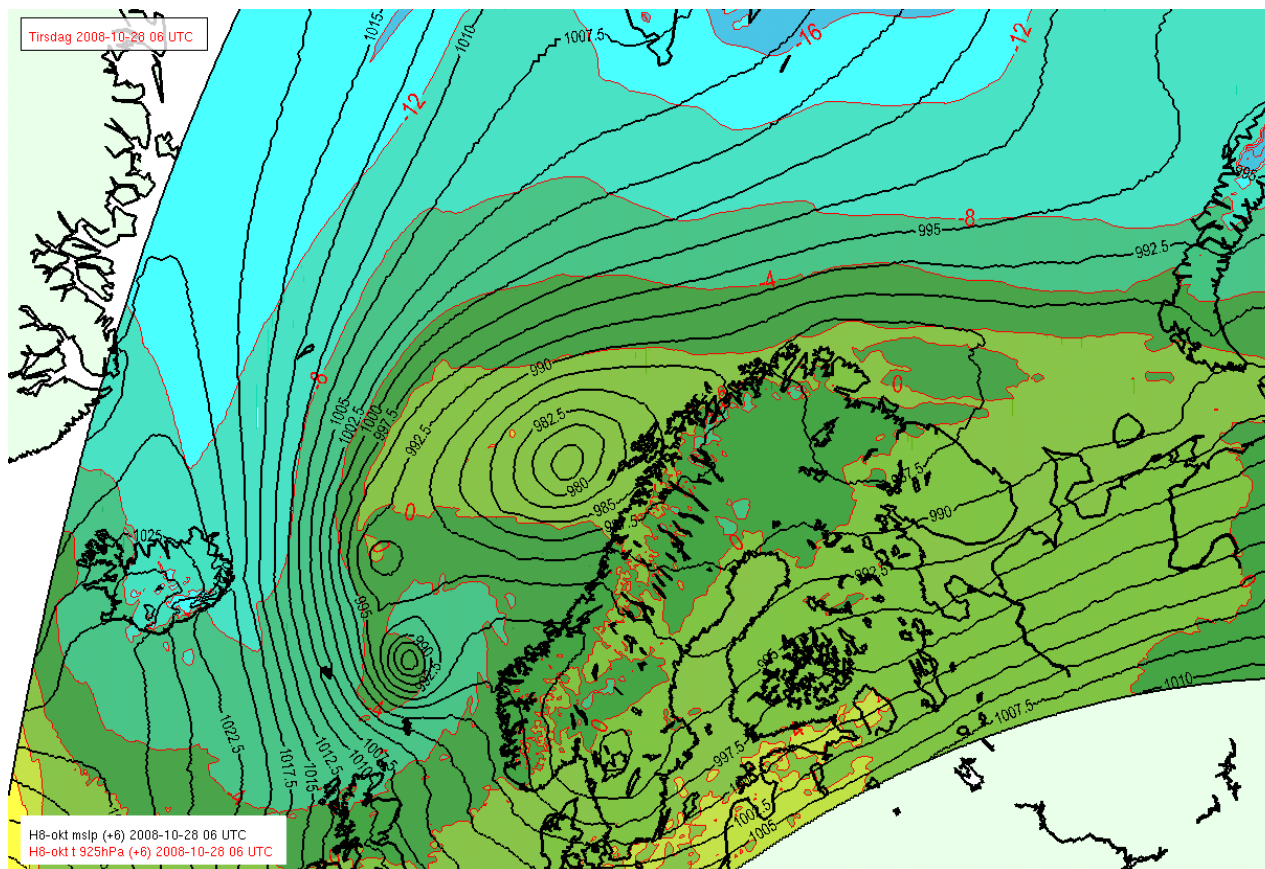


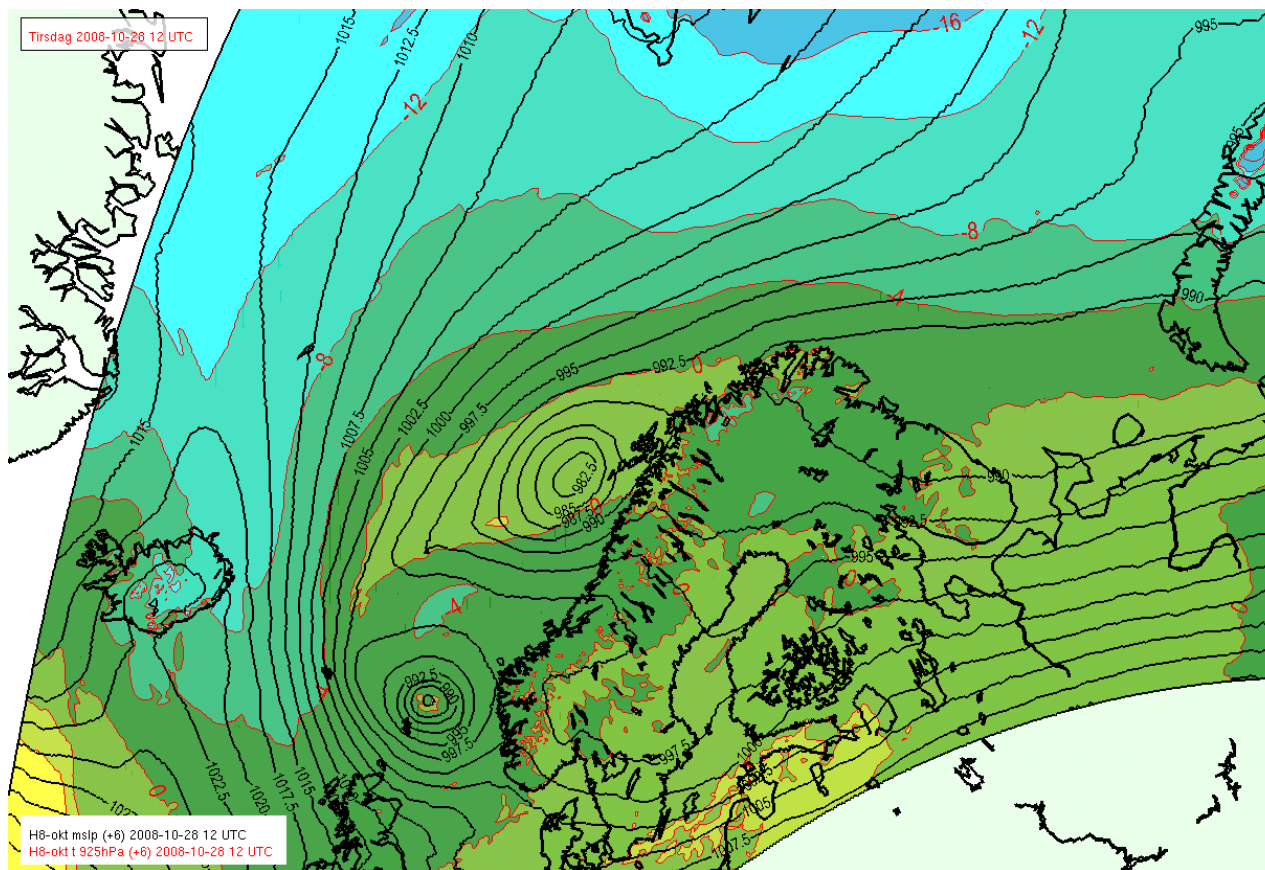












Baroclinic developments



1a) perturbations on a basic flow

Geostrophic streamfunction:

$$\vec{v}_\psi = \vec{k} \times \nabla \psi, \quad \zeta = \nabla^2 \psi, \quad \psi = \Phi / f$$

Quasi-geostrophic vorticity equation:

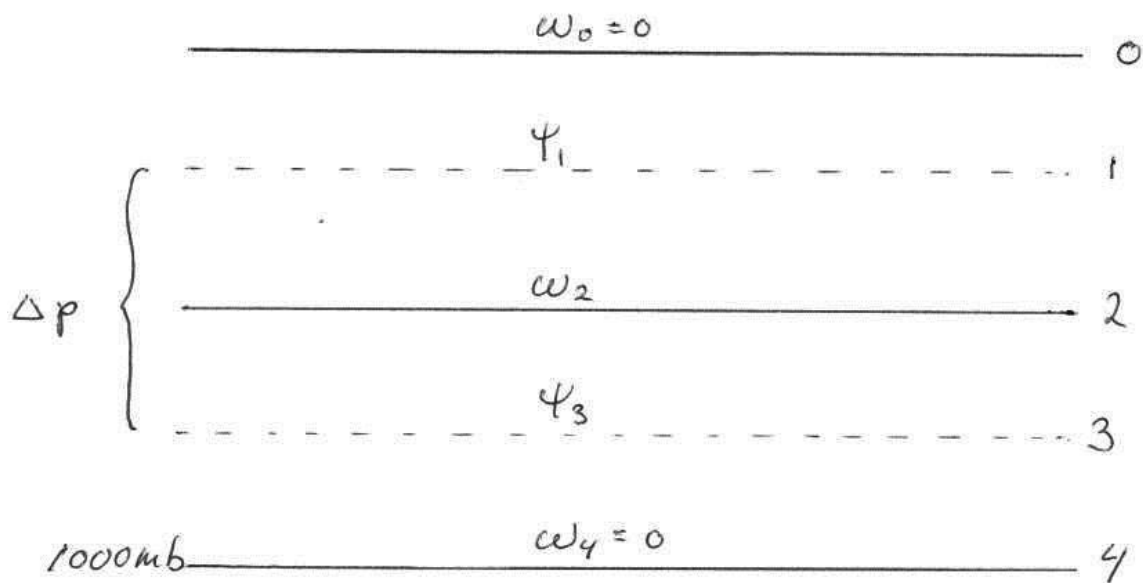
$$\frac{\partial}{\partial t} \nabla^2 \psi + \vec{v}_\psi \bullet \nabla (\nabla^2 \psi + f) - f \frac{\partial \omega}{\partial p} = 0$$

Thermodynamic energy equation:

$$\frac{\partial}{\partial t} \left(\frac{\partial \phi}{\partial p} \right) + \vec{v}_\psi \bullet \nabla \left(\frac{\partial \phi}{\partial p} \right) + \sigma \omega = 0$$



the two-parameter model





- In a traditional way we will assume that the flow consists of a basic state that depend linearly on y alone, plus perturbations that depend only on x and t . We also assume that the variation of f is only taken into account in the advection term of the vorticity equation and that a beta-plane approximation is used.
- We will however not assume the basic state to be zonal and chose the x -axis parallel with the thermal wind so that the y -axis points towards colder air.

$$\psi_1 = -U_1 y + \psi_1'(x, t)$$

$$\psi_3 = -U_3 y + \psi_3'(x, t)$$

$$\omega_2 = \omega_2'(x, t)$$



The vorticity equation in level 1 and 3 becomes,

$$\left(\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x}\right) \frac{\partial^2 \psi'_1}{\partial x^2} + U_1 \beta \cos \alpha + \frac{\partial \psi'_1}{\partial x} \beta \sin \alpha - f \left(\frac{\partial \omega'}{\partial p}\right)_1 = 0$$

$$\left(\frac{\partial}{\partial t} + U_3 \frac{\partial}{\partial x}\right) \frac{\partial^2 \psi'_3}{\partial x^2} + U_3 \beta \cos \alpha + \frac{\partial \psi'_3}{\partial x} \beta \sin \alpha - f \left(\frac{\partial \omega'}{\partial p}\right)_3 = 0$$

α is the angle between the thermal wind and true north and

$$\beta = |\nabla f|$$

for a pure northerly/southerly basic state

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x}\right) \left(\frac{\partial^2 \psi'}{\partial x^2} + f\right) - f \left(\frac{\partial \omega'}{\partial p}\right) = 0$$



Further assumptions

Ignore the “beta-effect”

$$\beta = 0$$

define

$$\psi_m = (\psi'_1 + \psi'_3) / 2, \quad \psi_T = (\psi'_1 - \psi'_3) / 2$$

$$U_m = (U_1 + U_3) / 2, \quad U_T = (U_1 - U_3) / 2$$



ω_2 is derived from the thermodynamic energy equation

$$\left(\frac{\partial}{\partial t} + U_m \frac{\partial}{\partial x}\right)(\psi'_1 - \psi'_3) - U_T \frac{\partial}{\partial x}(\psi'_1 + \psi'_3) = \frac{\sigma \Delta p}{f} \omega_2$$

adding and subtracting and utilizing the thermodynamic energy equation to eliminate ω_2

$$\left(\frac{\partial}{\partial t} + U_m \frac{\partial}{\partial x}\right)\left(\frac{\partial^2 \psi_T}{\partial x^2} - 2\lambda^2 \psi_T\right) + U_T \frac{\partial}{\partial x}\left(\frac{\partial^2 \psi_m}{\partial x^2} + 2\lambda^2 \psi_m\right) = 0$$

$$\left(\frac{\partial}{\partial t} + U_m \frac{\partial}{\partial x}\right)\frac{\partial^2 \psi_m}{\partial x^2} + U_T \frac{\partial}{\partial x}\left(\frac{\partial^2 \psi_T}{\partial x^2}\right) = 0$$

$$\lambda^2 = \frac{f^2}{\sigma(\Delta p)^2}$$



wavelike perturbations

$$\psi_m = A e^{ik(x-ct)}, \quad \psi_T = B e^{ik(x-ct)}$$

$$ik^3 ((c - U_m)A - ik^3 U_T B) = 0$$

$$k U_T (k^2 - 2\lambda^2)A - ik((c - U_m)(k^2 + 2\lambda^2))B = 0$$

the determinant has to vanish for non-trivial solutions

$$c = U_m \pm \sqrt{\frac{U_T^2 (k^2 - 2\lambda^2)}{(k^2 + 2\lambda^2)}}$$

$$\lambda^2 = \frac{f^2}{\sigma(\Delta p)^2}$$



$k^2 < 2\lambda^2$ (c will have an imaginary part and the wave is unstable)

$$L = 2\pi / k$$

$$L_c = \Delta p \pi \sqrt{2\sigma} / f \quad (\text{critical wavelength})$$

typical middle latitude conditions - ($L_c \sim 3000$ km)

typical polar conditions - ($L_c \sim 600$ km)

because of low tropopause, ~ 600 hPa

small static stability), \sim half its extratropical value

far north \Rightarrow larger f

growth rate

$$\alpha = k c_i = k \sqrt{\frac{U_T^2 (2\lambda^2 - k^2)}{(k^2 + 2\lambda^2)}}$$



Growth rate maximum, $\frac{\partial \alpha}{\partial k} = 0$

$$k^2 = 2\sqrt{2}\lambda^2 - 2\lambda^2$$

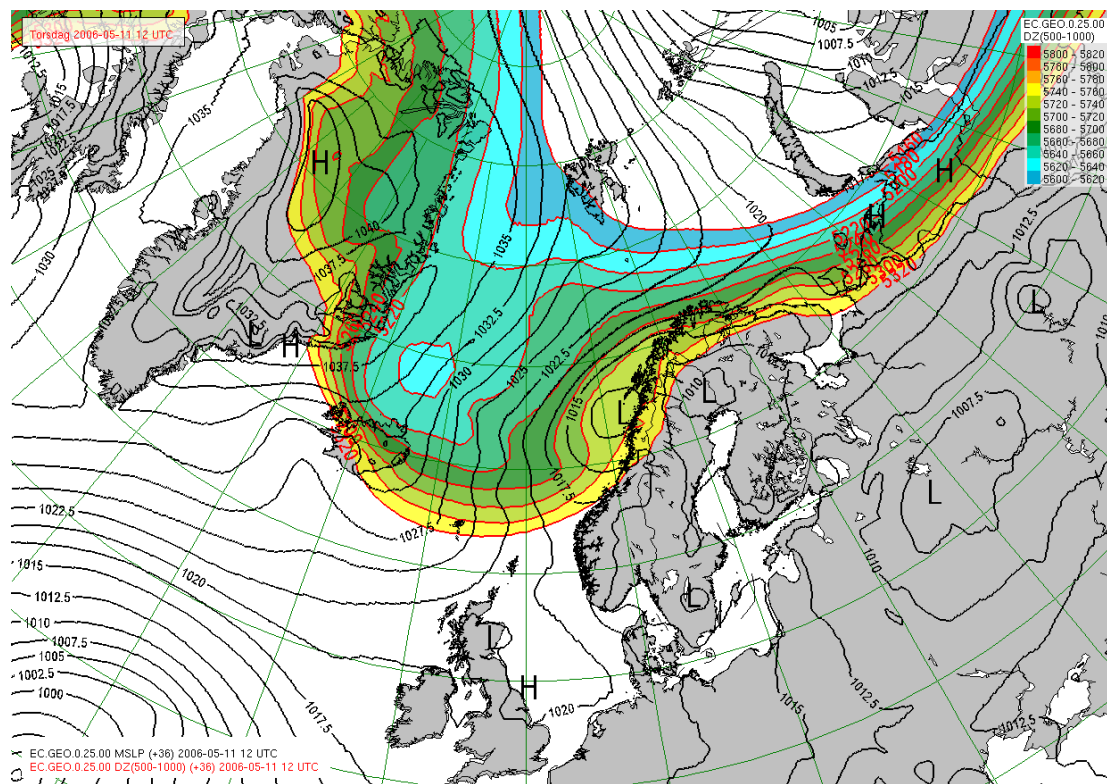
$$L^* = L_c \sqrt{1 + \sqrt{2}} \approx 1.55 \cdot L_c$$

Using the same numbers as before, this wavelength is 930 km, and reducing the stability further (another 50%) gives $L^*=650$ km.



Mesoscale cyclone embedded in large scale (synoptic scale) cyclone (background vorticity)

Discussed by Økland (1987)





We will again assume that the flow consists of a basic state and a perturbation

$$\psi_1 = -\Psi_1(y) + \psi_1'(x, t)$$

$$\psi_3 = -\Psi_3(y) + \psi_3'(x, t)$$

$$U_{1,3} = -\frac{\partial \Psi_{1,3}}{\partial y}$$

But the basic state (which is still assume stationary) has a horizontal shear, i.e. vorticity

$$Z_{1,3} = -\frac{\partial U_{1,3}}{\partial y} = \text{const}$$



The perturbation equations for vorticity now become (without beta-effect)

$$\left(\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x}\right) \frac{\partial^2 \psi'_1}{\partial x^2} - (f + Z) \left(\frac{\partial \omega'}{\partial p}\right)_1 = 0$$

$$\left(\frac{\partial}{\partial t} + U_3 \frac{\partial}{\partial x}\right) \frac{\partial^2 \psi'_3}{\partial x^2} - (f + Z) \left(\frac{\partial \omega'}{\partial p}\right)_3 = 0$$

the energy equation is unchanged,

λ^2 changes to,

$$\lambda^2 = \frac{f(f + Z)}{\sigma(\Delta p)^2}$$



critical wavelength

$$L_c = \Delta p \pi \sqrt{2\sigma} / \sqrt{f(f + Z)}$$

Using background vorticity $2 \cdot 10^{-4} \text{ s}^{-1}$, we find instability for wavelengths longer than $L_c \sim 300 \text{ km}$ without changing the other parameters, that maximum growth for wavelength: $L^* \sim 450 \text{ km}$

e-folding time: $\tau = \frac{1}{k c_i}$

e-folding time for the wave with the largest growth rate is found to be ~ 10 hours for $Z=0$ assuming $U_T = 10 \text{ ms}^{-1}/2$, which is considerably shorter than for typical synoptic scale cyclone at middle latitudes. In the case of ambient vorticity ($Z = 2 \cdot 10^{-4} \text{ s}^{-1}$), it's even shorter (~ 5 hours).



The wave speed (in the absence of the ‘beta-effect’) is,

$$c = U_m \pm \sqrt{\frac{U_T^2 (k^2 - 2\lambda^2)}{(k^2 + 2\lambda^2)}}$$

$$c = U_m \pm i \sqrt{\frac{U_T^2 (2\lambda^2 - k^2)}{(k^2 + 2\lambda^2)}} \equiv c_R + i c_i$$

The growing wave moves with the speed of the mean basic state. In the case of a reversed shear flow, this means a southerly direction, in the same direction as the surface wind, but with a slower speed than the surface flow itself.



An equivalent (but continuous) method was developed by Eady.

The Eady problem may also be cast in the more accurate geostrophic momentum approximation. There are, however, no fundamental differences between the results



Change of frame of reference

Define:

$$x' = x - Vt; \quad t' = t; \quad u' = u - V$$

Material derivative

$$\frac{D'}{dt'} = \frac{\partial}{\partial t'} + u' \frac{\partial}{\partial x'} = \frac{D}{dt}$$

Navier Stokes equations are unchanged, but the primitive equations are not (The Coriolis force is not a Newtonian force and is not Galilean invariant)



But does it matter for the linear baroclinic adiabatic stability analysis?

NO!

Because vorticity and thermodynamic equations are unchanged (the stretching term $f \frac{\partial \omega}{\partial p}$ comes from $-f \nabla \cdot \mathbf{v}$ and the transformation velocity is constant)



What about friction?

- Holopainen showed in the early sixties that the effect of friction in a similar analysis has the effect of slowing down and reducing the growth rate.
- Is this also true if cast in transformed coordinates?
- Friction is a «real» force in contrast to the Coriolis force and should be Galilean invariant.

$$\tau = -\rho C \bar{v}^2$$



But the way it is parameterized, it is
NOT

$$\tau = \rho C \bar{v}^2$$

$$\tau = \rho C (\bar{v}' + \vec{V}_T)^2 = \rho C (\bar{v}'^2 + 2\bar{v}' \bullet \vec{V}_T + \vec{V}_T^2)$$



Potential vorticity may be created or destroyed due to diabatic heating and friction

$$q = \frac{1}{\rho} \boldsymbol{\eta}_a \cdot \nabla \theta,$$

$$\frac{dq}{dt} = \frac{1}{\rho} (\boldsymbol{\eta}_a \cdot \nabla \dot{\theta} + \nabla \times \mathbf{F} \cdot \nabla \theta).$$



friction

$$\left(\frac{dq}{dt}\right)_{\text{fric}} = \frac{1}{\rho} \left[\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \frac{\partial \theta}{\partial z} + \left(\frac{\partial F_x}{\partial z} \right) \left(\frac{\partial \theta}{\partial y} \right) - \left(\frac{\partial F_y}{\partial z} \right) \left(\frac{\partial \theta}{\partial x} \right) \right]$$



Cooper et al. (1992): frictional source (sink) of PV if the surface wind has a component opposite (same) the direction of the thermal wind

Giving:

- reversed shear => PV generation (due to friction)
- forward shear => PV destruction (due to friction)



Surface fluxes of heat

Beare (2007): PV destroyed proportional to heat flux at the top of the boundary layer

$$\left[\frac{DP}{Dt} \right] \approx -\frac{\xi_h H_s}{\rho_0 h^2}$$

Note that PV at the surface will be destroyed in regions of high upward fluxes, i.e. in the cold air

Baroclinic developments

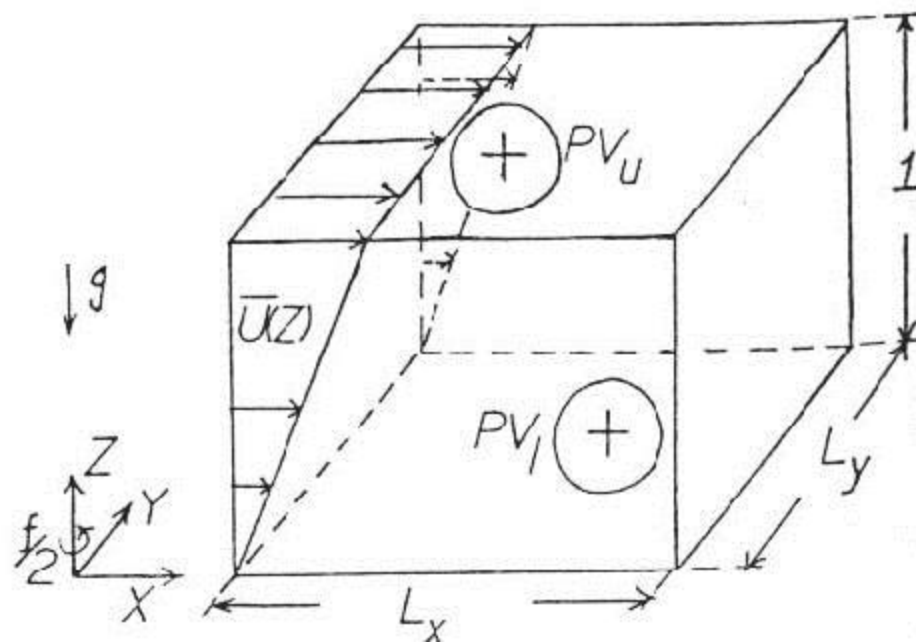


b) Finite disturbances as an initial value problem (Montgomery and Farrel, 1992)

- Interacting upper and lower potential vorticity structures
- Using a nonlinear geostrophic momentum model
- Including moist physics (imposed moist neutrality for ascending parcels)
 - (note: conditional neutrality doubles the growth rate, Emanuel et al. 1987; Joly and Thorpe 1989)
- Initial condition is upper-level trough positioned upstream of a surface disturbance
- Surface disturbance;
 - cold air outbreak, surface fronts, occluded cyclone; vorticity debris to the rear of a cold front, high surface air temperatures, etc.....

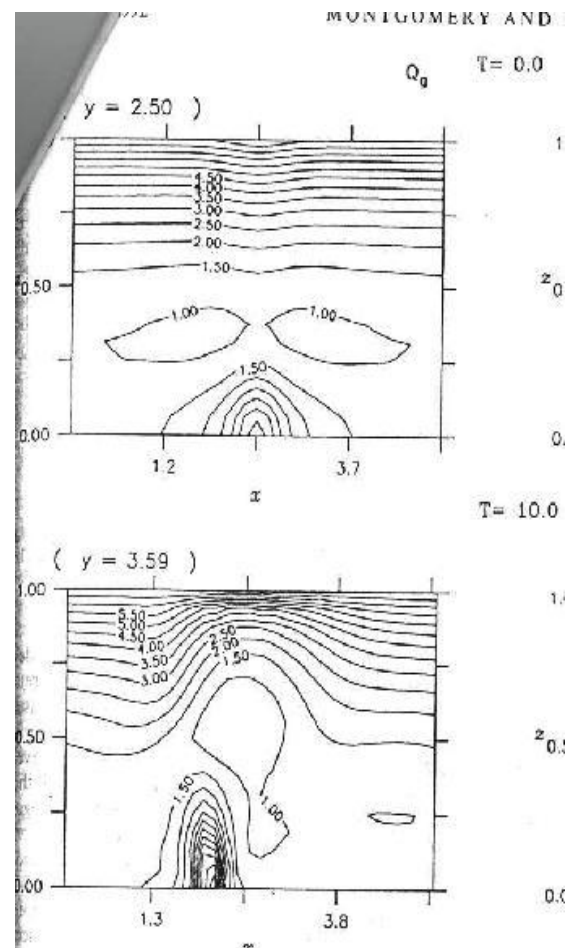
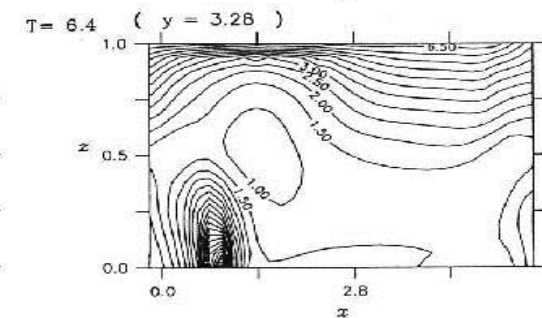
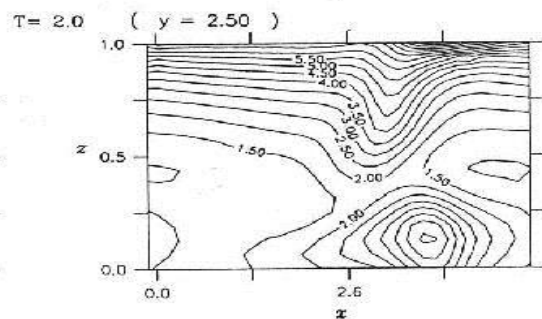
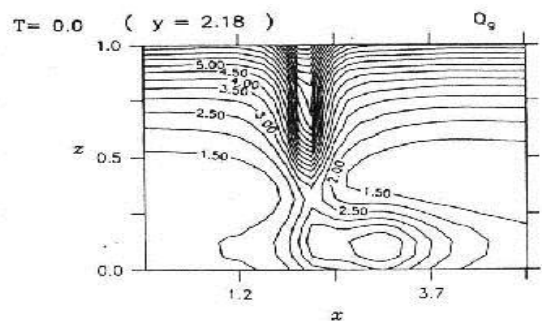


The Montgomery and Ferrel model





with upper level PV anomaly (left)
without upper level PV anomaly (right)





Secondary development - diabatic destabilization

Associated with the generation of potential vorticity at low levels

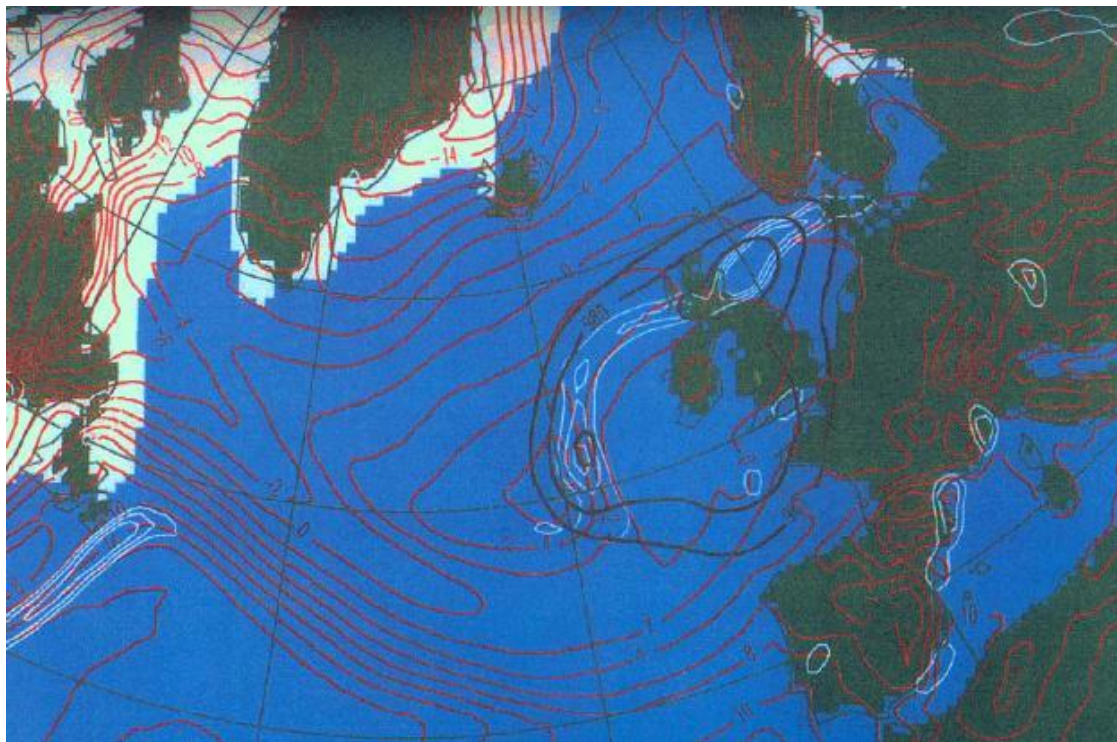
Conceptual model:

1. Initial growing phase due to interaction with mobile upper level trough; creation of large values of PV at low levels
2. Maintenance phase (diabatic intensification)



c) Barotropic instability

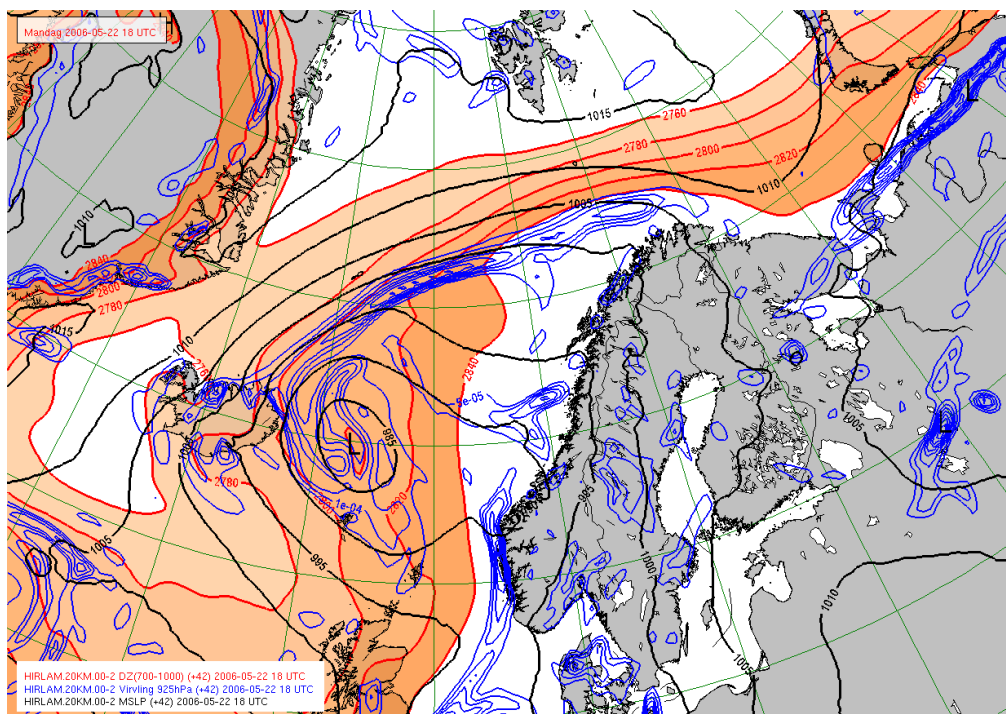
- The basic flow has rarely vanishing or uniform vorticity





3) Barotropic instability (contd.)

- A necessary condition for barotropic instability is that the vorticity has a maximum within the flow

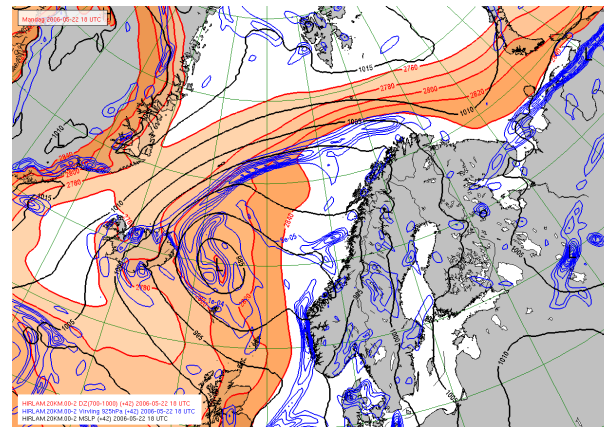




4) Baroclinic instability (contd.)

The Raleigh Theorem:

If the horizontal temperature gradient vanishes at the lower boundary: then a necessary condition for baroclinic instability is that the potential vorticity has a maximum within the flow



(see e.g. Holton, 4th ed, 2004, p 256)

2. Diabatic developments



a) CISK (Conditional Instability of the Second Kind)

Bratseth (1985) - "A note on CISK in polar air masses"

Boussinesq equations linearized around a basic state at rest

$$\frac{\partial u}{\partial t} = fv - \frac{\partial \phi}{\partial x}$$

$$\frac{\partial v}{\partial t} = -fu - \frac{\partial \phi}{\partial y}$$

$$0 = \frac{g}{\theta_0} \theta - \frac{\partial \phi}{\partial z}$$

$$\frac{\partial \theta}{\partial t} = -\frac{\theta_0}{g} N^2 w + \frac{\theta_0}{c_p T_0} Q$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

solve for w



$$\left(\frac{\partial^2}{\partial t^2} + f^2\right) \frac{\partial^2}{\partial z^2} w + N^2 \nabla^2 w = \frac{g}{c_p T_0} \nabla^2 Q$$

In the small amplitude limit, Bratseth disregards the time derivative compared to f^2 . Inserting a Fourier component

$$Q = \hat{Q}(x, t) \cos kx \cos ly$$

$$w = \hat{w}(x, t) \cos kx \cos ly$$

$$\frac{\partial^2}{\partial z^2} \hat{w} - \frac{N^2 K^2}{f^2} \hat{w} = -\frac{g}{c_p T_0 f^2} K^2 \hat{Q}, \quad K^2 = k^2 + l^2$$



Assume further that the heat released in the column is equal to the latent heat that enters through the top of the boundary layer.

$$\int_0^{\infty} Q \rho dz \equiv Q_0 \bar{\rho} (H_2 - H_1) = \rho_s q_s L w_s$$

An expression for w at the surface is derived from Ekman pumping

$$w_s = D \zeta_s$$

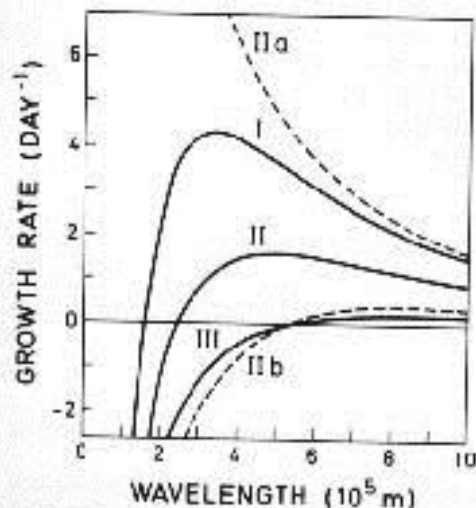


Fig. 2. Growth rates as a function of wavelength for five different vertical distributions of heating. (I) $H_1 = 500$ m, $H_2 = 1500$ m. (II) $H_1 = 500$ m, $H_2 = 2500$ m. (III) $H_1 = 500$ m, $H_2 = 4500$ m. (IIa) $H_1 = 0$ m, $H_2 = 2000$ m. (IIb) $H_1 = 1000$ m, $H_2 = 3000$ m.

Tellus 3^A (1985), 5

with $\sigma = 0$ ($\partial w / \partial z = 0$ for $z = 0$). Clearly, strong stretching at $z = 0$ is favoured if a large part of the heating takes place at a low level. This fits well with

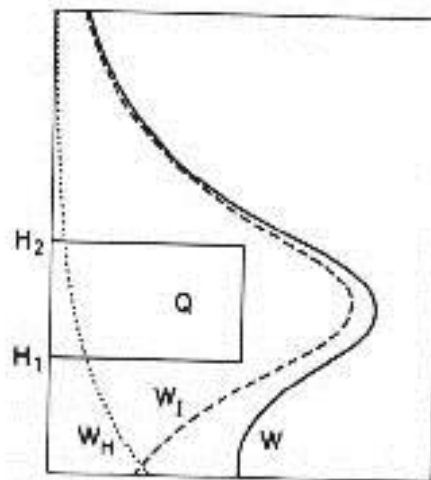


Fig. 3. Example of vertical velocity amplitude as a function of height.



Taking the time derivative and utilizing the vorticity equation at the surface gives at $z=0$,

$$\frac{\partial w}{\partial t} = D \frac{\partial \zeta}{\partial t} = Df \frac{\partial w}{\partial z}$$

Inserting the results for w into this expression, Bratseth finds that the solution grows exponentially. The growth rate give an e-folding time of the order of 10 hours if the heating is released close to ground

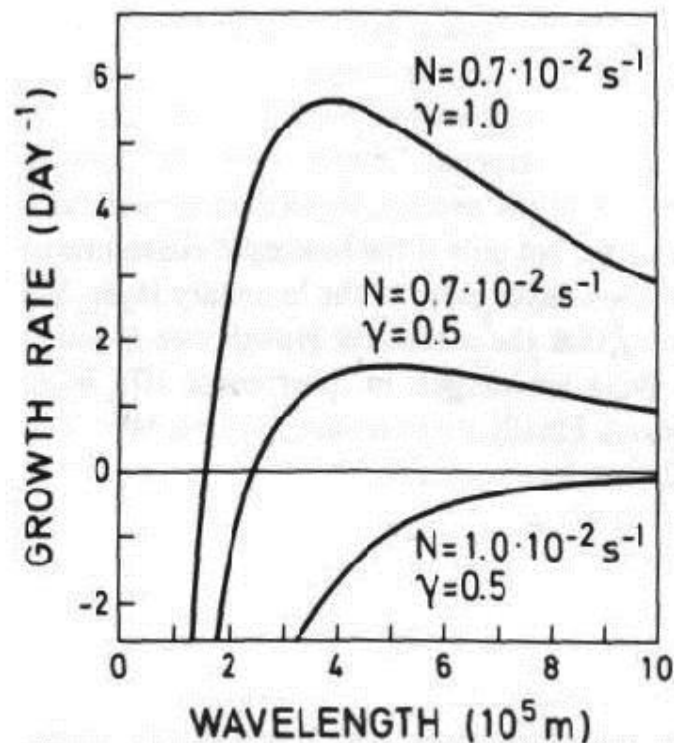


Fig. 4. Growth rate as a function of wavelength in case II (Fig. 2) for different values of N and γ .



b) WISHE instability theory

Emanuel has derived the WISHE instability theory (Wind Induced Surface Heat Exchange) where the growth of the cyclone is only due to heat acquired at the surface and released in the ascending motion.

Emanuel and Rotunno (1989) showed by using an axisymmetric model that intense hurricanes can develop in Arctic environments according to the theory, but that an initial disturbance was necessary.

The growth rate is however smaller than for observed polar lows.



WISHE theory applied on a PL simulation

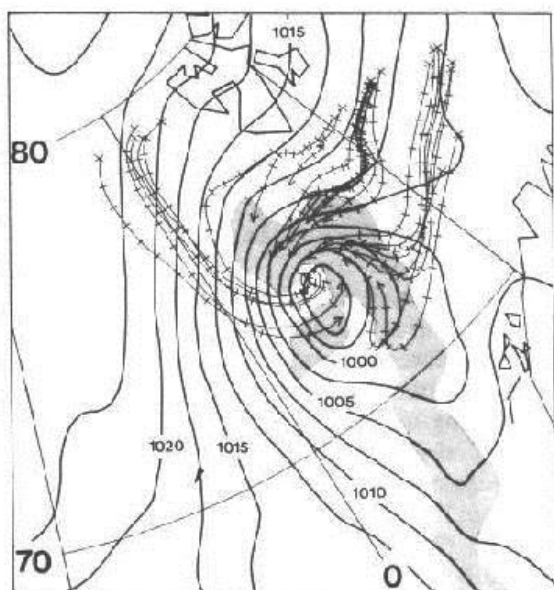


Fig. 13. Mean sea level pressure at contour intervals of 2.5 hPa (solid lines) and 24-h surface wind trajectories ending at 1200 GMT 26 January 1987, based on initialized analyses and short-range forecast winds (+2 and +4 h) from the lowest model layer (≈ 40 m) with temporal resolution 2 h. Shaded area signifies 2-h accumulated precipitation at the end of the period.

Nordeng (Tellus, 1990)



3) Baroclinicity as a precursor for diabatic growth (setting up favourable conditions for latent heat release)

Haugen (1985) showed by running a primitive equation model in a channel in a reversed shear flow that baroclinic waves developed as expected according to theory. The simulations were dry, but friction were included.

Interestingly he also found that during the course of the development, an area of conditionally instability evolved in the region where one would expect the cloud sheet (comma shape) to be found.

The instability region deepened as the low intensified.



The adiabatic energy equation

$$\frac{1}{\theta} \frac{D\theta}{Dt} = 0$$

$$\frac{\partial}{\partial t} \ln \theta + \vec{v} \cdot \nabla \ln \theta + w \frac{\partial}{\partial z} \ln \theta = 0$$

derivation with respect to z gives,

$$\frac{\partial}{\partial t} \frac{\partial \ln \theta}{\partial z} + \vec{v} \cdot \nabla \frac{\partial \ln \theta}{\partial z} + \frac{\partial \vec{v}}{\partial z} \cdot \nabla \ln \theta + w \frac{\partial}{\partial z} \frac{\partial \ln \theta}{\partial z} + \frac{\partial w}{\partial z} \frac{\partial \ln \theta}{\partial z} = 0$$

or

$$\frac{D}{Dt} N^2 = -\frac{g}{\theta} \frac{\partial \vec{v}}{\partial z} \cdot \nabla \theta - \frac{\partial w}{\partial z} N^2$$

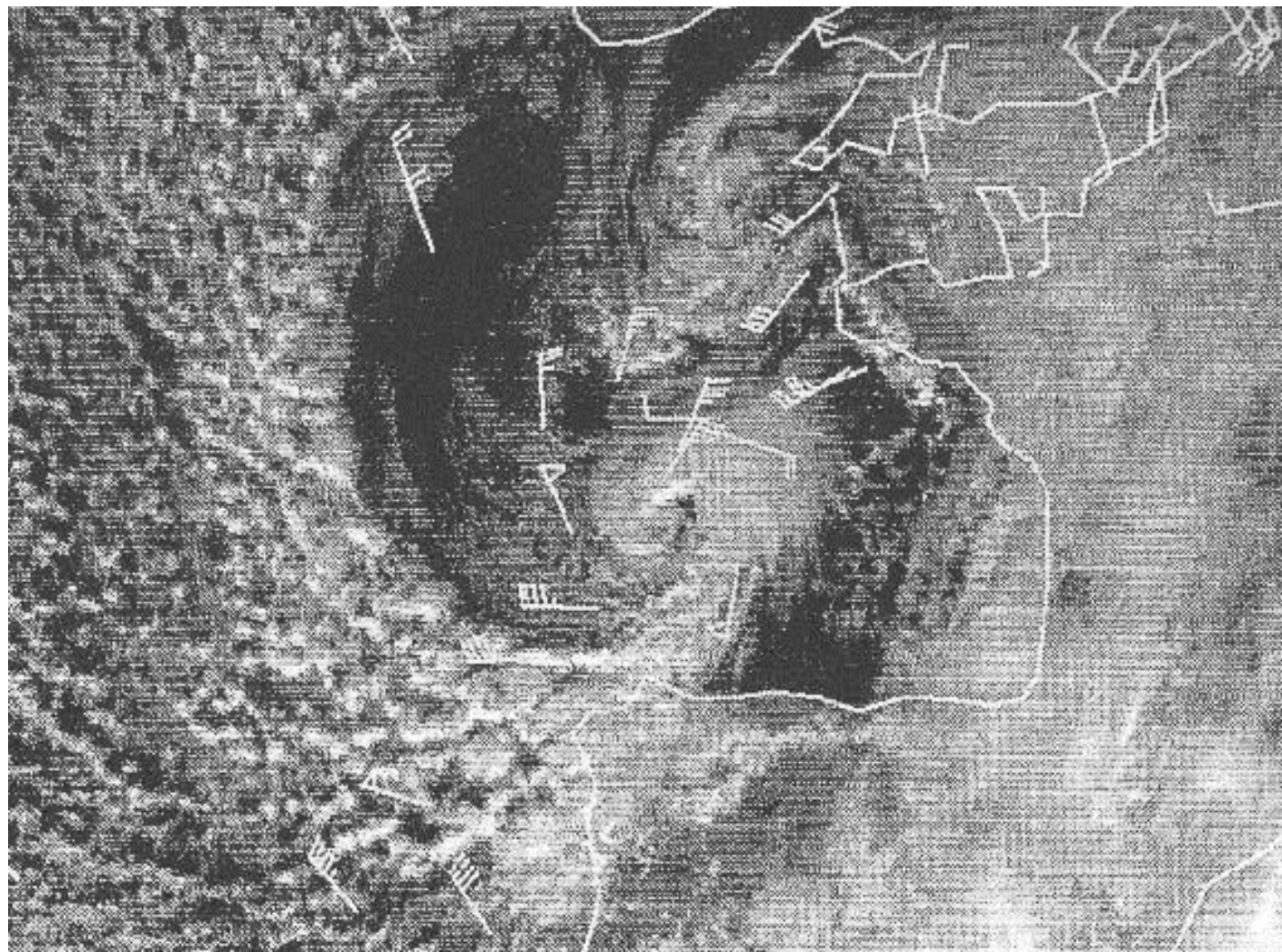
showing that a parcel loses buoyancy where there is low level convergence from vortex stretching (last term is negative). The first term gives destabilisation ahead of the low (cold front) and stabilisation in the rear (the warm front) for reversed shear flow

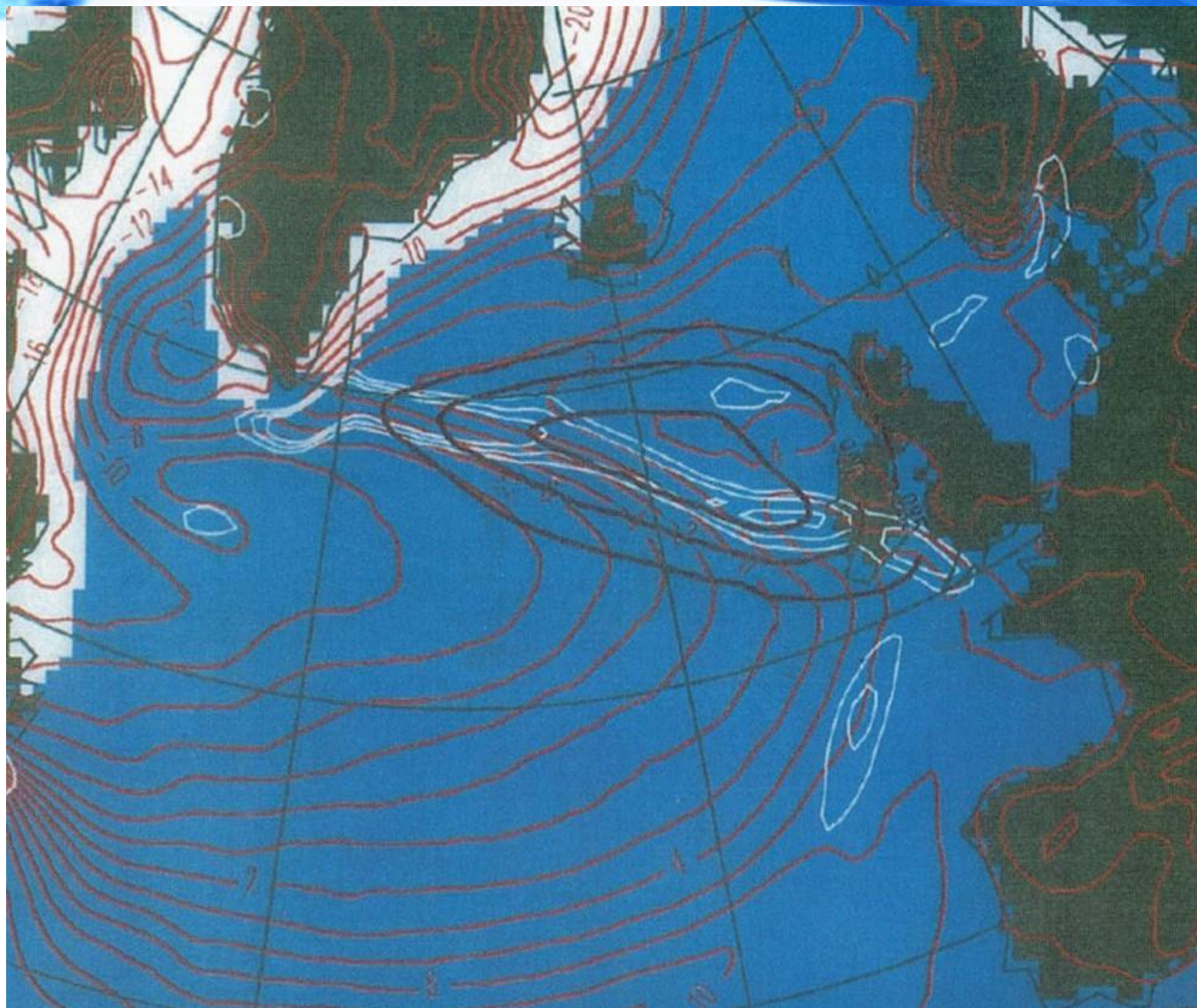


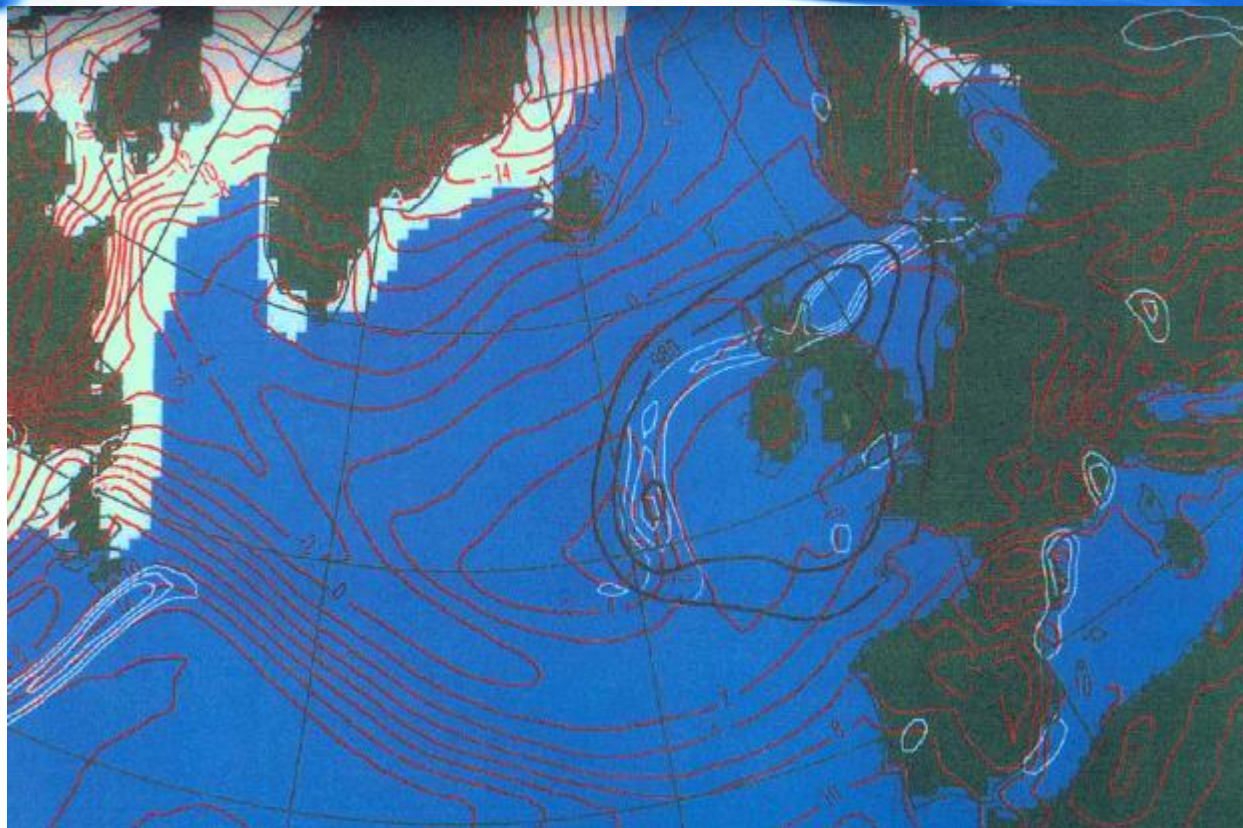
Why are reversed shear flow cases seldom explosive, maybe except for a short initial phase even when the low has the right size for growth?

Answer?

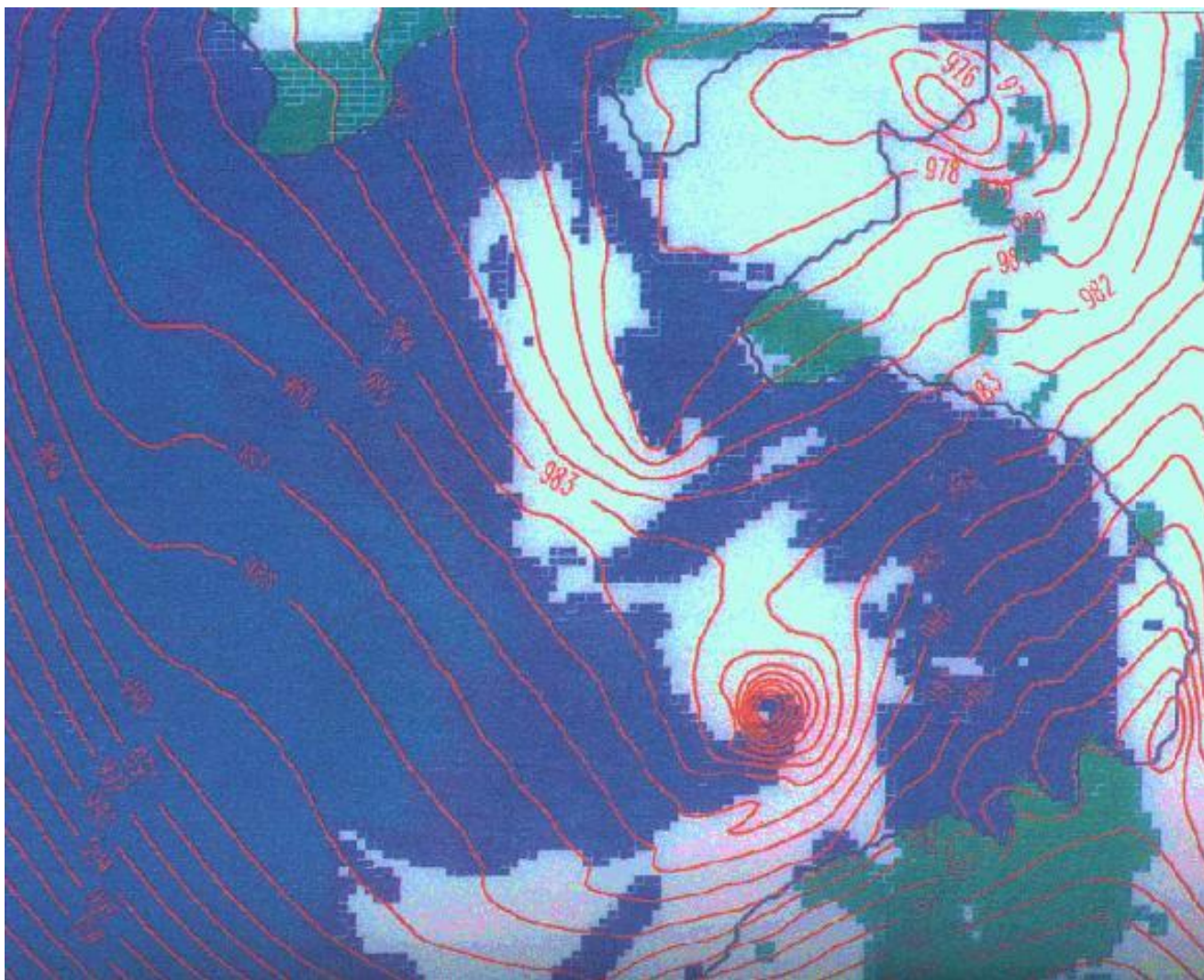
- the upper air disturbance in basic flow moves in the opposite direction to low level disturbance, => short time for contact, less probable that locking may occur
- weak upper flow => i.e. none or weak upper air disturbance

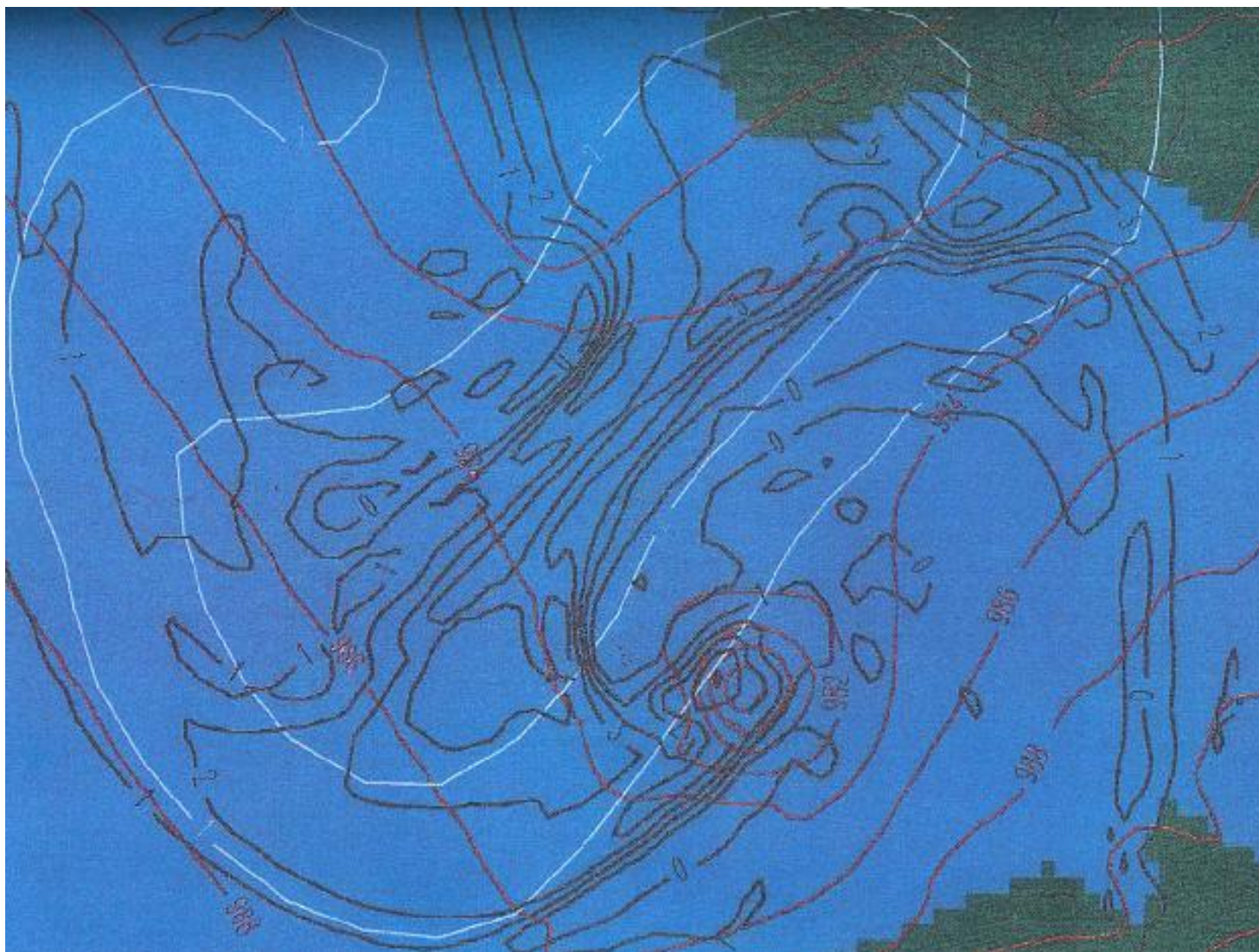


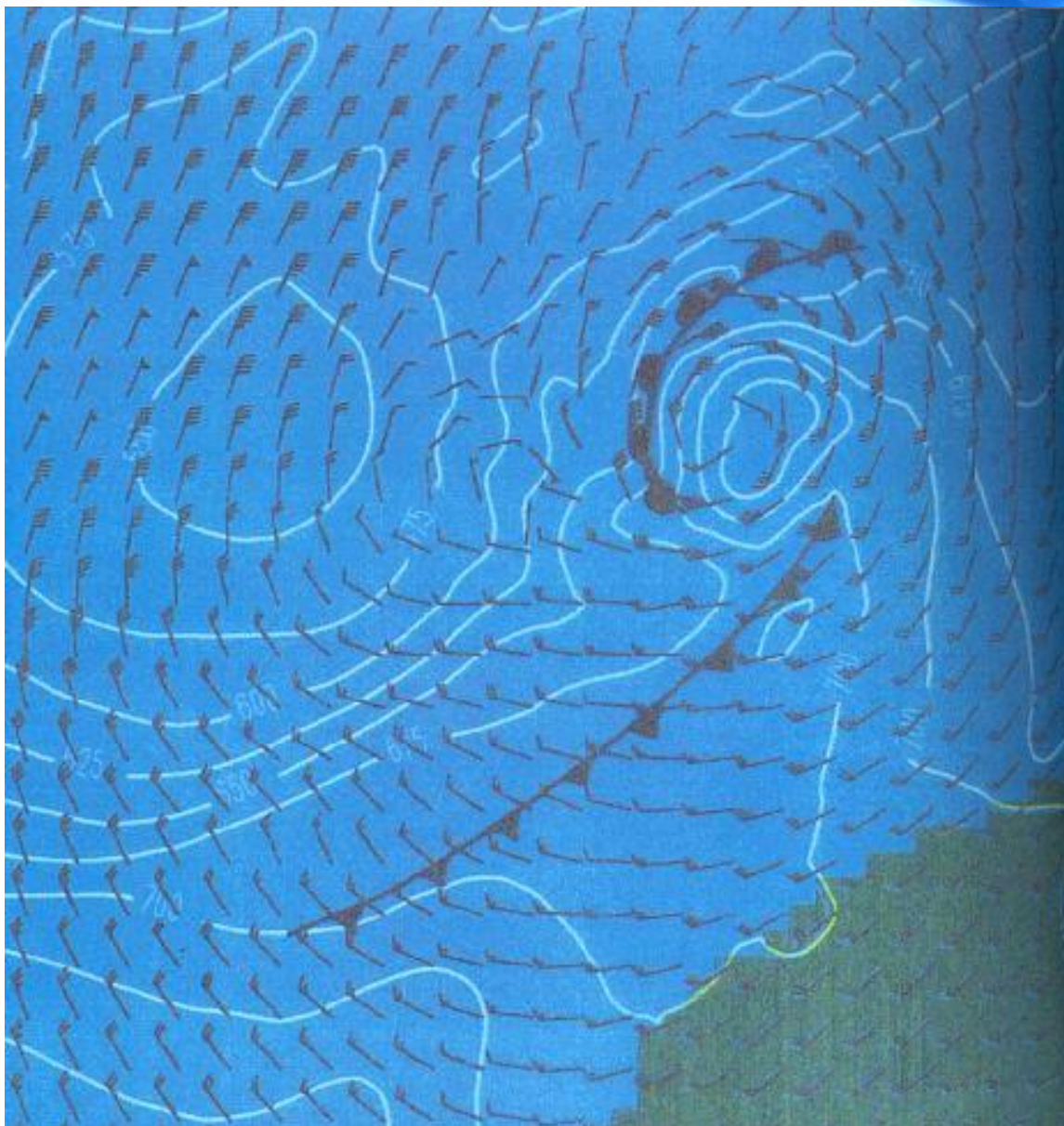




Red lines are mean temperature between 700 and 1000 hPa. Some contours of mslp (black) and some contours of relative vorticity at 1000 hPa are also shown. Date 5th January 1994







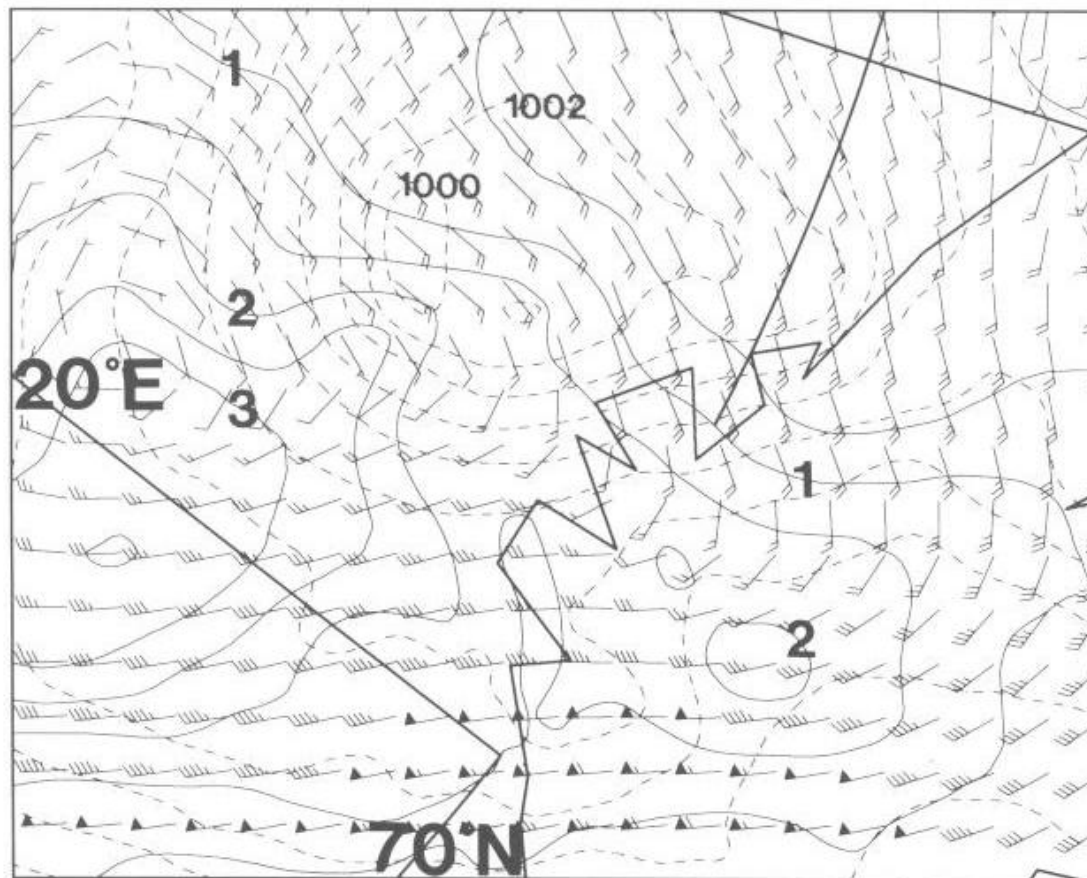
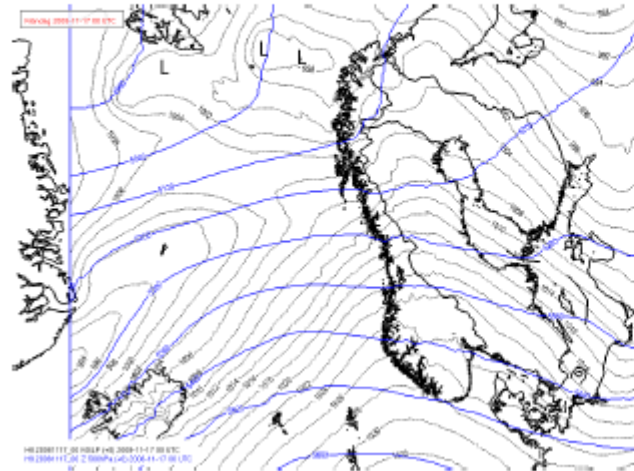


Fig. 12. Full lines are potential vorticity at the 278 K isentropic surface from 1200 UTC 26 February 1987, at contour intervals of 0.5 potential vorticity units (PVU) ($1 \text{ PVU} = 10^{-6} \text{ m}^2 \text{ s}^{-1} \text{ K kg}^{-1}$). Also shown are horizontal winds at the 278 K surface valid at the same time, a flag is 25 ms^{-1} , a full barb is 5 ms^{-1} and a half barb is 2.5 ms^{-1} . Dash-dot lines are mean sea level pressure at contour intervals of 2 hPa valid at 1800 UTC 26 February (i.e., 6 h later).

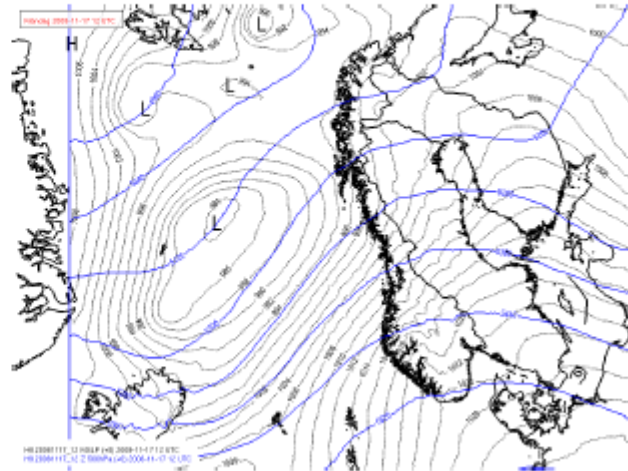


Cold air outbreaks creates conditions favourable for PL developments, but not polar lows (I have never seen a PL in the middle of a cold air outbreak)

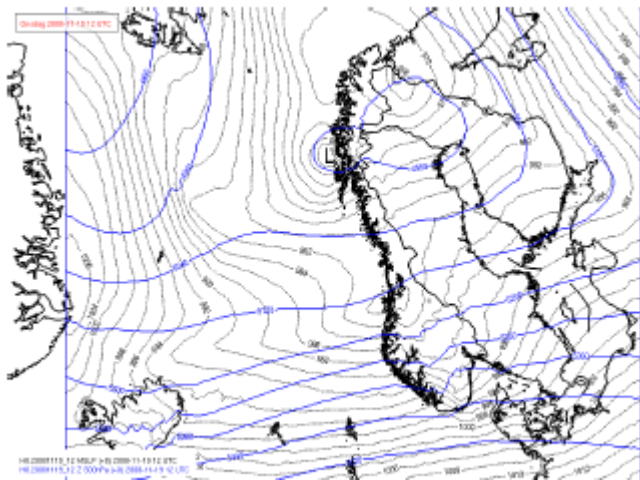
Example from recent paper in Quarterly Journal (Nordeng and Røsting, 2011): A polar low named Vera



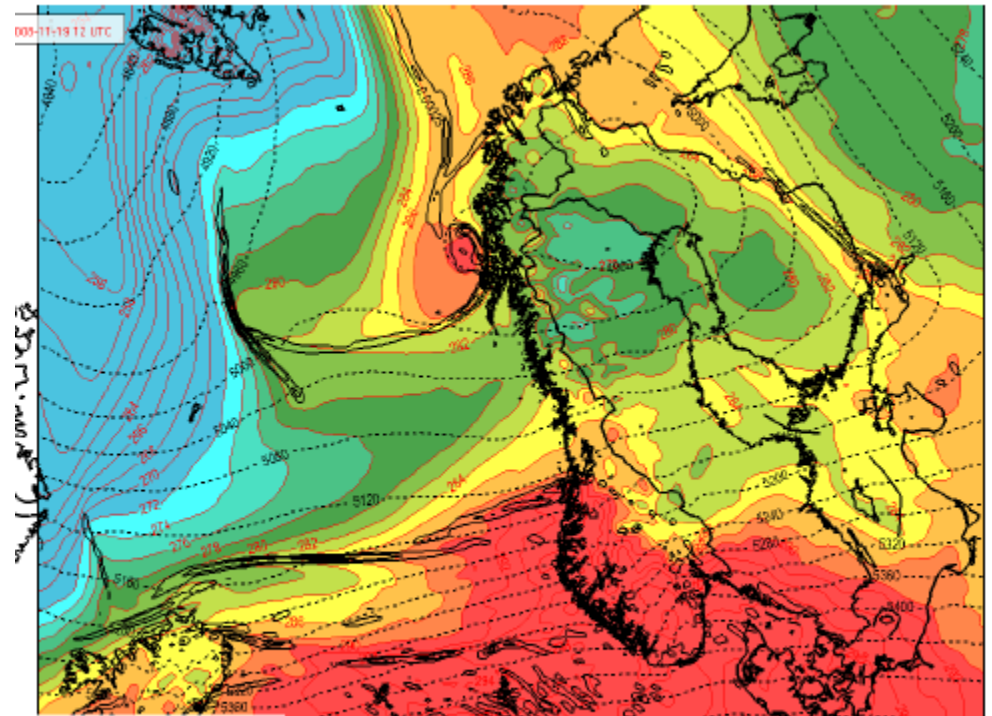
17-11-00UTC



17-11-12UTC



19-11-12UTC



119_12 THE SSTPa (+0) 2006-11-19 12 UTC
119_12 Viewing SSTPa (+0) 2006-11-19 12 UTC
119_12 THE SSTPa (+0) 2006-11-19 12 UTC

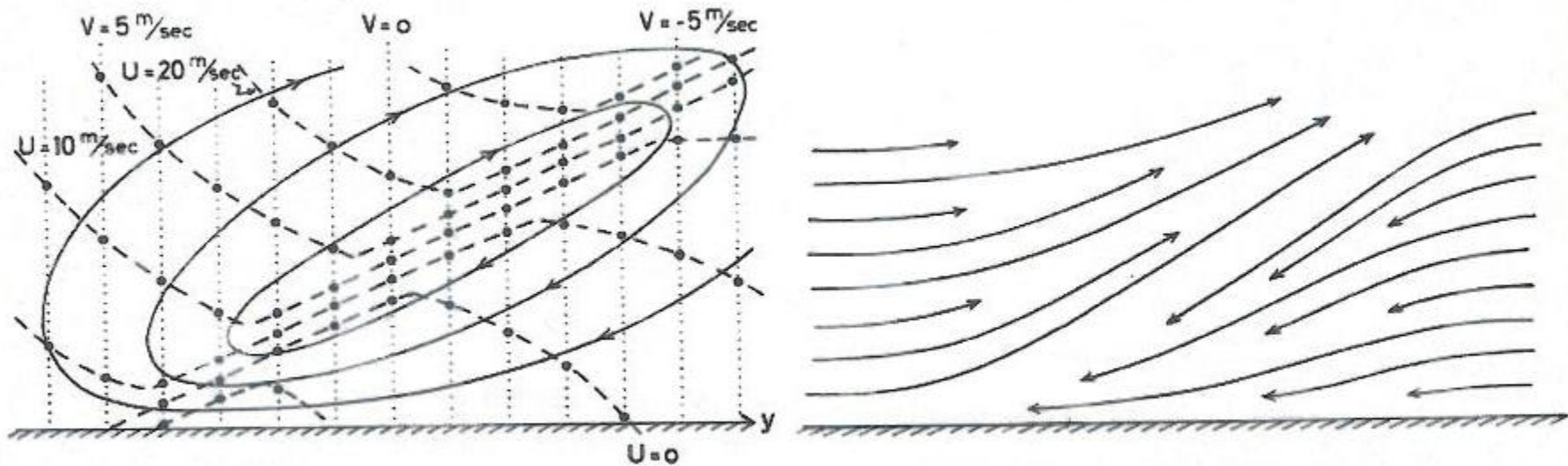


Fig. 2. Transverse motion in an idealized frontal zone where $\partial V/\partial y < 0$, $\partial V/\partial p = 0$.

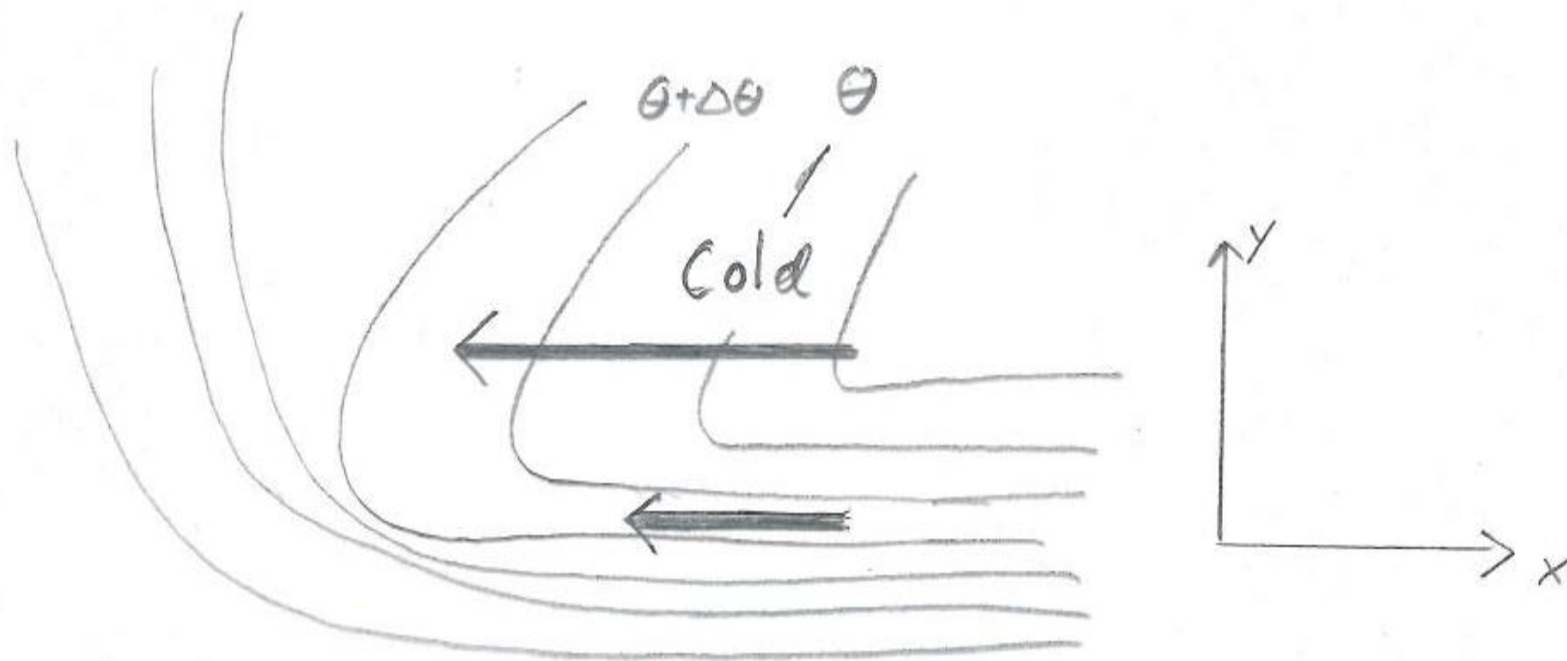
a. Dashed lines: U -isotachs. Dotted lines: V -isotachs. Solid lines: Streamlines of transverse non-geostrophic circulation.

b. Streamlines of convergent total (geostrophic and non-geostrophic) transverse motion relative to the motion of the front.

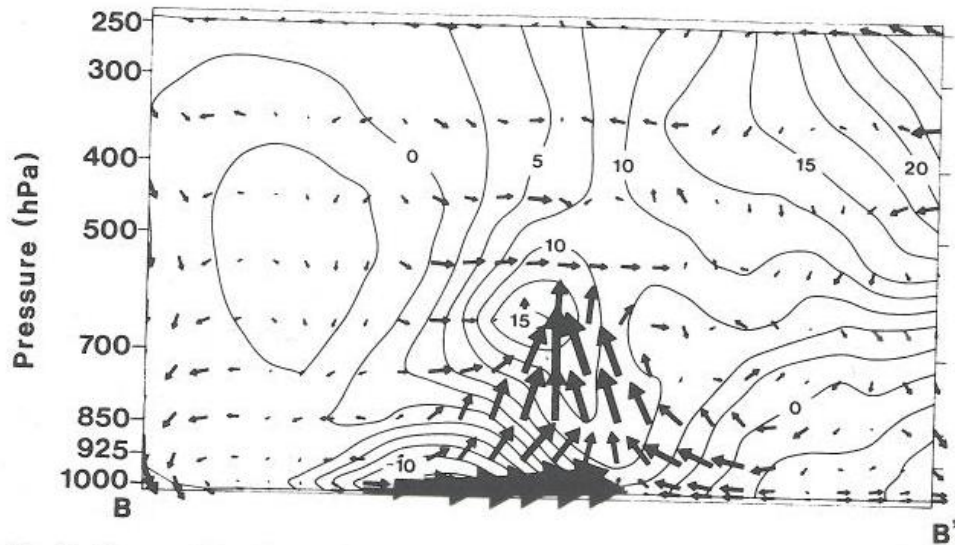
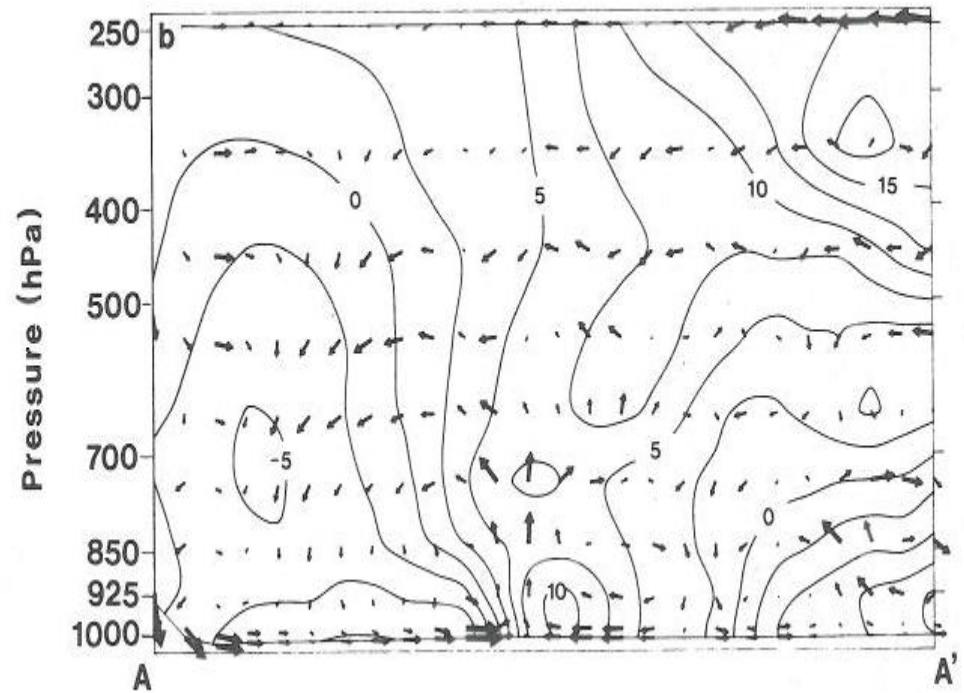
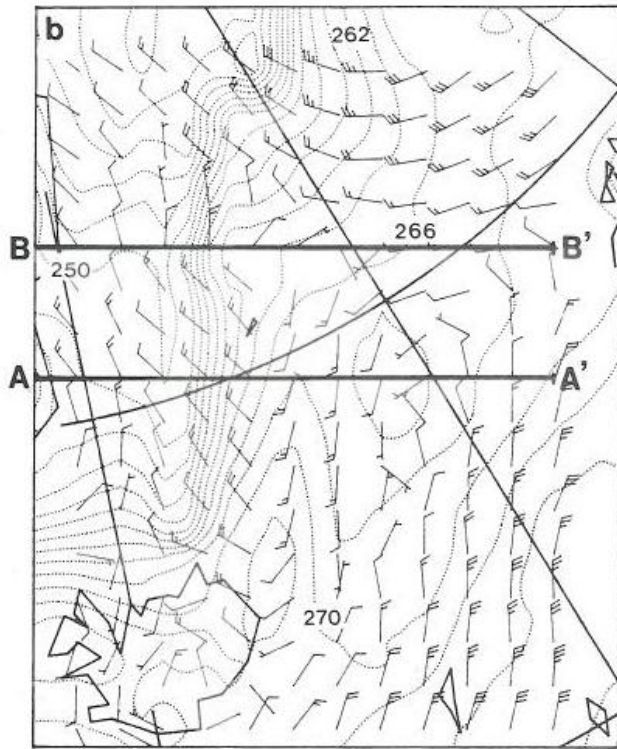
Forcing of transverse circulation by,

$$Q = -2\gamma \left(\frac{\partial U}{\partial x} \frac{\partial \theta}{\partial y} - \frac{\partial U}{\partial y} \frac{\partial \theta}{\partial x} \right), \quad \gamma = \frac{R}{f p_0} \left(\frac{p_0}{p} \right)^{c_v/c_p}$$

Direct circulation for positive values of Q



$$\begin{aligned} \frac{\partial U}{\partial y} < 0 \\ \frac{\partial \theta}{\partial x} < 0 \end{aligned} \Rightarrow Q > 0$$



Nordeng (Tellus, 1990)



Summary/Conclusion

- Shed some light on the role of the (mysterious) reversed shear and its relevance for PL developments
- But it is nothing in the dynamics of the system that should give preference to this as compared to forward shear cases for strong developments
- It is rather where and how the low level baroclinicity develops (in which environments) that counts.

Thank you for your attention

